

4.1

Representing Percents

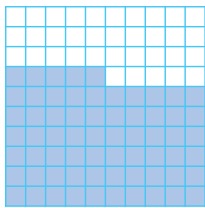
Focus on...

After this lesson, you will be able to...


- show percents that are between 0% and 1%
- show percents that are greater than 100%
- show percents involving fractions

percent

- means *out of 100*
- another name for hundredths
- 65% means 65 out of 100 or $\frac{65}{100}$ or 0.65.



Materials

- hundred grids 
- coloured pencils

People often read nutrition labels on food products to determine the **percent** of the recommended daily value (RDV) of nutrients the food contains. By reading these labels you can make wiser food choices to help maintain a healthy lifestyle.

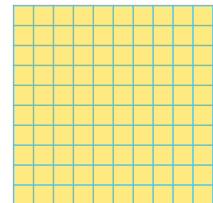
The nutrition label on a certain brand of grape juice says that one 250-mL glass of juice contains 130% of the RDV of Vitamin C, 2% of the RDV of iron, and 1% of the RDV of sodium. Half a glass would contain 65% of the RDV of Vitamin C, 1% of the RDV of iron, and $\frac{1}{2}\%$ of the RDV of sodium. You have seen how to represent a percent like 65% on a grid. How might you use grids to represent 130% or $\frac{1}{2}\%$?



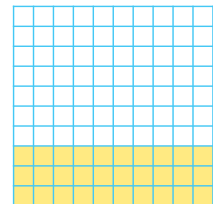
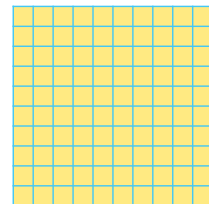
Explore the Math

How can you represent percents on a grid?

1. **a)** The hundred grid shows 100%. How many squares are shaded?

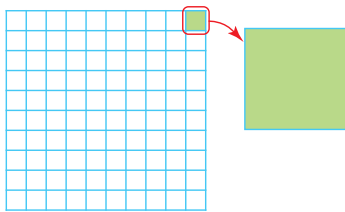


- b)** Explain how the following diagram shows 130%.



- c)** Shade hundred grids to show 350%. How many squares did you shade?

2. a) Shade a hundred grid to show half of 100%. How many squares did you shade? What percent of the whole grid do the shaded squares represent?
 - b) Shade a hundred grid to show half of your answer to part a). How many squares did you shade? What percent of the whole grid do the shaded squares represent?
 - c) Shade a hundred grid to show half of your answer to part b). How many squares did you shade? What percent of the whole grid do the shaded squares represent?
 - d) How does the type of number represented by the percent value in part c) differ from the types of numbers in parts a) and b)? Explain why.
3. The circled square represents 1% on the hundred grid shown.



- a) What fraction of the enlarged square would you need to shade to show half of 1%? What percent of the whole grid would your shaded portion represent?
- b) What fraction of a 1% square would you need to shade to represent $\frac{3}{4}$ %?
- c) What fraction of a 1% square would you need to shade to represent 0.37%?

Reflect on Your Findings

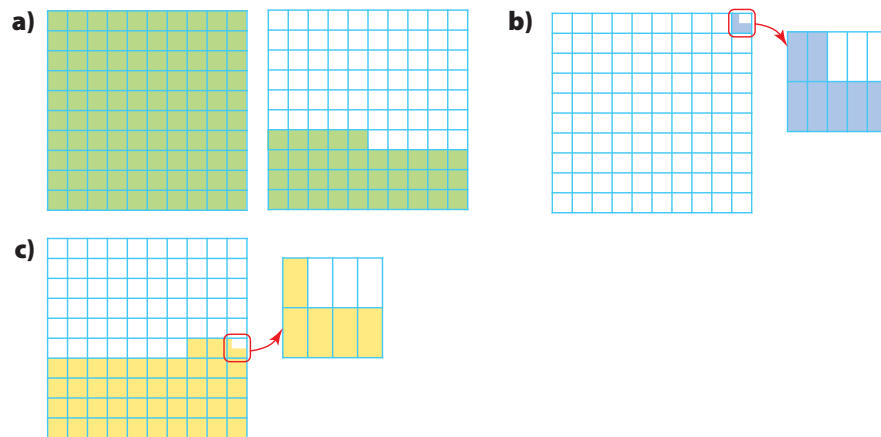
4. Describe how to use grids to represent the following types of percent values.
 - a) percents greater than 100%
 - b) percents between 0% and 1%
 - c) percents containing a mixed number greater than 1%

History Link

In Roman times, the term *centurion* was used to describe an officer in the Roman Legion who was in charge of 100 soldiers. There was one centurion *per cent*, meaning there was one centurion per 100 soldiers. What other English words do you know that include *cent*?

Example 1: Determine the Percent Represented on a Grid

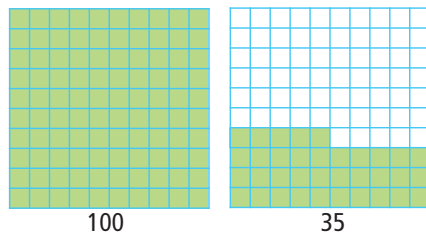
One completely shaded grid represents 100%. What percent does each diagram represent?



Solution

- a) Each grid is divided into 100 squares. A completely shaded grid represents 100%.

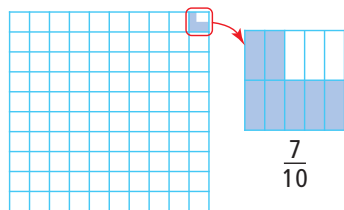
The first grid is completely shaded. There are 100 squares shaded. In the second grid, there are three full rows of ten shaded and five squares shaded in the fourth row. There are 35 squares shaded. There are a total of 135 squares shaded.



This diagram represents 135%.

- b) Since a portion of only one square of a hundred grid is shaded, the percent represented is between 0% and 1%. You can *zoom in* on the partially shaded square and count the number of shaded parts. The enlarged diagram shows seven out of a total of ten parts shaded.

The shading represents $\frac{7}{10}$ or 0.7 of 1% of the whole diagram.



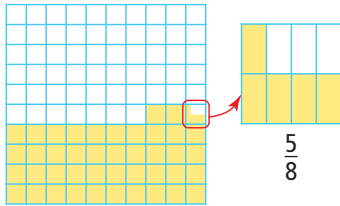
The diagram represents $\frac{7}{10}$ % or 0.7%.

- c) The diagram shows 42 squares shaded plus a portion of another square.

You can *zoom in* on the partially shaded square to determine the fraction that is shaded.

The enlarged diagram shows $\frac{5}{8}$ shaded.

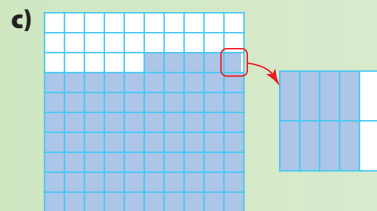
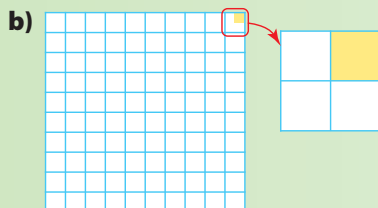
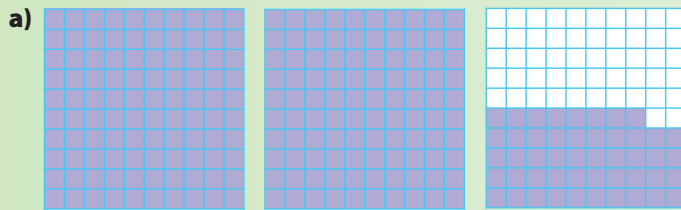
The shading represents $\frac{5}{8}$ of 1% of the whole diagram.



The diagram represents $42\frac{5}{8}\%$.

Show You Know

One completely shaded grid represents 100%. What percent does each diagram represent?



Example 2: Represent Percents on a Grid

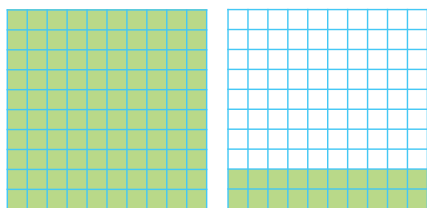
Represent the percent in each statement on a grid.

- An orange juice container shows that one 250-mL serving contains 120% of the recommended daily value of Vitamin C.
- A significant portion of the world's fresh water is found in Canada, but Canada has only 0.5% of the world's population.
- A credit card company charges an interest rate of $18\frac{3}{4}\%$ on unpaid balances.

Solution

- Since 120% is greater than 100%, more than one hundred grid is needed.

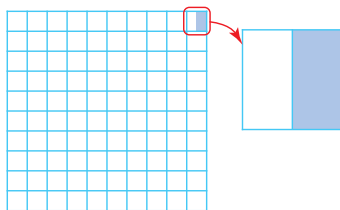
You can represent 100% by completely shading one grid. You can represent 20% by shading 20 squares of a second hundred grid.



fractional percent

- a percent that includes a portion of a percent, such as $\frac{1}{2}\%$, 0.42%, $7\frac{3}{8}\%$, $125\frac{3}{4}\%$, 4.5%

- 0.5% is a **fractional percent** between 0% and 1%. *Zoom in* on one square of a hundred grid. Since 0.5 represents $\frac{1}{2}$, divide the enlarged square into two equal sections. Shade one of the two sections.

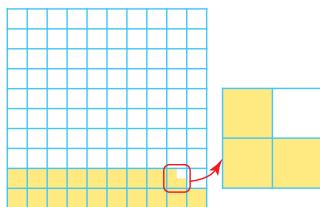


- $18\frac{3}{4}\%$ is a fractional percent between 1% and 100%.

Use one hundred grid.

Shade 18 squares to represent 18%.

Shade $\frac{3}{4}$ of another square to represent $\frac{3}{4}\%$.



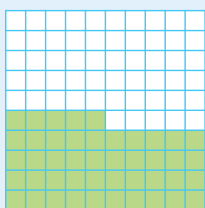
Show You Know

Represent each percent on a grid.

- a) 180% b) 0.6% c) $12\frac{3}{8}\%$

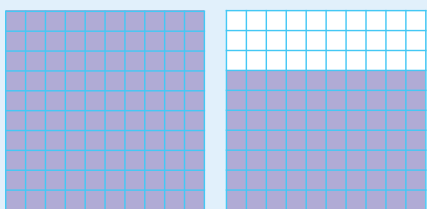
Key Ideas

- To represent a percent, you can shade squares on a grid of 100 squares called a hundred grid. One completely shaded grid represents 100%.



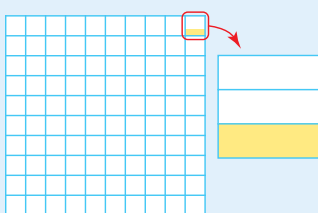
45%

- To represent a percent greater than 100%, shade more than one grid.



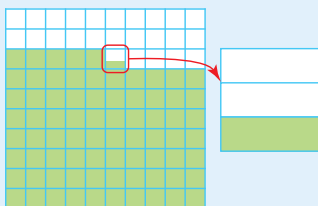
170%

- To represent a fractional percent between 0% and 1%, shade part of one square.



$\frac{1}{3}\%$

- To represent a fractional percent greater than 1%, shade squares from a hundred grid to show the whole number and part of one square from the grid to show the fraction.



$75\frac{1}{3}\%$

Communicate the Ideas

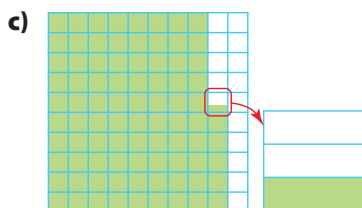
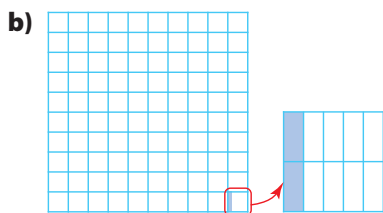
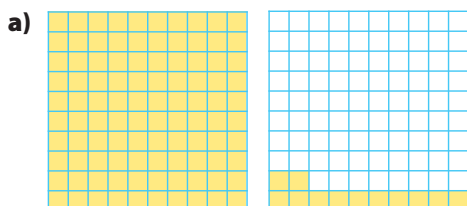
- Use hundred grids and words to describe the similarities and differences between a percent less than 1%, a percent between 1% and 100%, and a percent greater than 100%.
- You are asked to show a classmate how to use hundred grids to show 243%. How do you explain which squares need shading?
 - Explain how you would represent $25\frac{1}{4}\%$ on a grid.
- Shindi commented to a friend that “some percents would be easier to show if we shaded the parts that were not included in the percent.” Explain what she means. Which percents are easier to show using Shindi’s method? Why?

Check Your Understanding

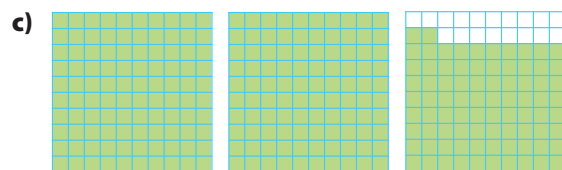
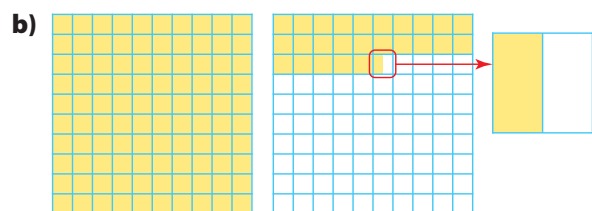
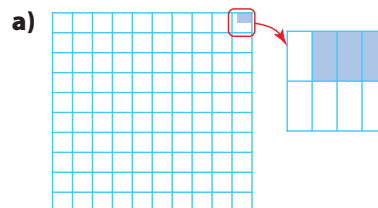
Practise

For help with #4 and #5, refer to Example 1 on pages 124–125.

4. One full grid represents 100%. What percent does each diagram represent?



5. What percent is represented by each diagram if a completely shaded grid represents 100%?



For help with #6 and #7, refer to Example 2 on page 126.

6. Represent each percent on a grid.
- 125%
 - $10\frac{1}{2}\%$
 - 0.4%
 - 262%
 - $\frac{7}{8}\%$
 - 45.6%

7. Represent the percent in each statement on a grid.
- Attendance at the fall fair increased by 3.2% this year.
 - The average mass of a Singapura cat is about 0.13% of the mass of a Siberian tiger.
 - The length of the Yukon River is about 230% of the length of the Fraser River.
8. How many hundred grids are needed to show each of the following percents?
- 300%
 - 466%
 - 1200%

Apply

- Give two examples where a percent greater than 100% might be found in everyday life.
- Why might a scientist studying water pollution work with percents less than one?
- The land area of Alberta is about 113% of the land area of Saskatchewan. Use hundred grids to show how the land area of Alberta compares with the land area of Saskatchewan.
- A 250-mL glass of milk contains 30% of the recommended daily value of calcium. Use a hundred grid to show how many glasses of milk you would need to drink to get 100% of the daily value of calcium.

Extend

- Use a calculator to convert $\frac{1}{3}$ to a decimal. How could $\frac{1}{3}\%$ be shown using a hundred grid?
 - Why are percents involving repeating decimals sometimes difficult to show on a hundred grid?
- If 200 squares were used instead of 100 squares to represent 100%, how would you show 0.25%?
 - If 400 squares were used instead of 100 squares to represent 100%, how would you show 0.75%?
- Show how hundred grid(s) could be used to represent a very small percent, such as 0.000 0125%.
- Suppose one large square represents 100%. The square is divided into smaller equal-sized pieces.
 - If there are 1000 pieces, what percent do 17 pieces represent?
 - If there are two large squares each divided into ten smaller pieces, what percent do 13 pieces represent?
 - If the large square is divided into eight smaller pieces, show how to represent $87\frac{1}{2}\%$ and $56\frac{1}{4}\%$.

MATH LINK

Use hundred grids to represent the following data.

97.5% of Earth's Water is Salt Water

2.5% of Earth's Water is Fresh Water

0.04% of Fresh Water Found in Earth's Atmosphere

0.007% of Fresh Water Accessible for Drinking Water

$\frac{3}{10}\%$ of Fresh Water Found in Lakes and Rivers

4.2

Fractions, Decimals, and Percents

Focus on...

After this lesson, you will be able to...

- convert between fractions, decimals, and percents



The period was not bad with 90% of the shots saved by the home team goalie. The second period saw 150% as many shots on goal, yet an amazing 0.9333 save performance held the home team in the game. But, how many ways can you spell disaster? In the third period, the home team goaltender let in two easy goals for a dismal $66\frac{2}{3}\%$ of shots on goal stopped.

Sports commentators often use statistics to report on the performance of a goalie. Commentators often change the way the information is presented to make it sound more interesting.

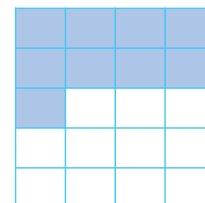
How did the sports commentator use the information from the following table in the report on the goalie's performance?

Goalie Statistics				
Period	Shots on Goal	Saves	Goals Against	Save Percent
1	10	9	1	90%
2	15	14	1	$93\frac{1}{3}\%$
3	6	4	2	$66\frac{2}{3}\%$

Explore the Math

How are percents related to fractions and decimals?

- What fraction of this figure is shaded?
 - Rewrite your fraction with a denominator of 100.
 - Express the fraction shaded as a decimal.
 - What percent of the figure is shaded?



- Suppose you want to shade one half as many sections as in #1. Show the area that will be shaded on a new diagram. How much of the diagram will you shade? Express your answer as a fraction, a decimal, and a percent.

3. Suppose you want to shade three times as many sections as in #1. If one large square represents one whole, how many squares will you need to draw to show this situation? How many squares will you shade? Express your answer as a fraction, a decimal, and a percent.

Reflect on Your Findings

4. a) How are the decimal, percent, and fraction representations of a number the same? How are they different?
 b) Which representations do you prefer to work with? Why?

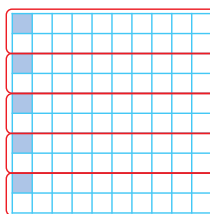
Example 1: Convert Fractions to Decimals and Percents

Convert each fraction to a decimal and a percent.

a) $\frac{1}{20}$ b) $\frac{71}{200}$ c) $\frac{9}{8}$

Solution

- a) Percent means out of 100. So, $\frac{1}{20} = \frac{x}{100}$.
 You could represent this using a hundred grid.
 5 of 100 squares are coloured.
 So, $\frac{1}{20} = \frac{5}{100}$. That is 5% or 0.05.
 Sometimes you interpret $\frac{1}{20}$ as $1 \div 20 = 0.05$.
 0.05 can be expressed as 5%.

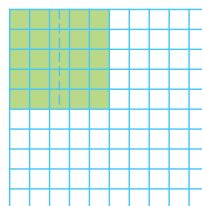


Is $\frac{1}{20}$ greater than or less than one whole? Will the percent be greater than or less than 100%? **ME**

- b) $\frac{71}{200} = \frac{x}{100}$
 $x = 35.5$ How do you know $x = 35.5$?
- That is 35.5% or 0.355.
 You could interpret $\frac{71}{200}$ as $71 \div 200 = 0.355$.
 0.355 can be expressed as 35.5%.

Is $\frac{71}{200}$ greater than or less than one whole? Will the percent be greater than or less than 100%? **ME**

- c) $\frac{9}{8}$ can be expressed as $\frac{8}{8} + \frac{1}{8} = 1 + \frac{1}{8}$.
 One whole represents 100%.
 You know that $\frac{1}{4}$ represents 25%. So, $\frac{1}{8}$ represents 12.5%.
 $\frac{9}{8}$ can be expressed as $100\% + 12.5\% = 112.5\%$.
 You could also interpret $\frac{9}{8}$ as $9 \div 8 = 1.125$.
 1.125 can be expressed as 112.5%.



Is $\frac{9}{8}$ greater than or less than one whole? Will the percent be greater than or less than 100%? **ME**

Show You Know

Convert each fraction to a decimal and a percent.

a) $\frac{3}{40}$

b) $\frac{171}{300}$

c) $\frac{88}{50}$

Example 2: Convert Decimals to Percents and Fractions

Convert each decimal to a percent and a fraction.

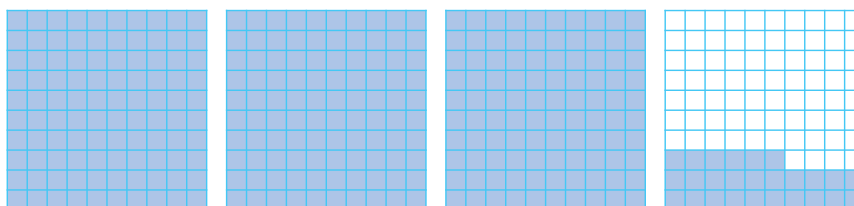
a) 3.26

b) 0.125

c) 0.0032

Solution

a) Use hundred grids.



3.26 = 3 full hundred grids plus 26 squares

That is $\frac{326}{100} = 326\%$.

$3.26 = 3\frac{26}{100}$ or $3\frac{13}{50}$.

Since 13 is a prime number, $3\frac{13}{50}$ is in lowest terms.

$\frac{326}{100}$ can also be expressed as $\frac{163}{50}$ or $3\frac{13}{50}$ in lowest terms.

What would you divide into both the numerator and denominator of $\frac{326}{100}$ to get $\frac{163}{50}$?

b) $0.125 = \frac{125}{1000}$ since the 5 is in the thousandths place.

0.125 can also be expressed as $\frac{12.5}{100}$ or 12.5%.

What factors of 125 divide evenly into 1000?

$\frac{125}{1000} = \frac{1}{8}$

How do you know $\frac{125}{1000}$ and $\frac{12.5}{100}$ are equivalent?

c) $0.0032 = \frac{32}{10\,000}$ since the 2 is in the ten thousandths place.

0.0032 can also be written as $\frac{0.32}{100}$ or 0.32%.

What factors of 32 divide evenly into 10 000?

$\frac{32}{10\,000} = \frac{2}{625}$

How do you know that $\frac{32}{10\,000}$ and $\frac{0.32}{100}$ are equivalent?

Show You Know

Convert each decimal to a percent and a fraction.

a) 0.0064

b) 0.268

c) 5.98

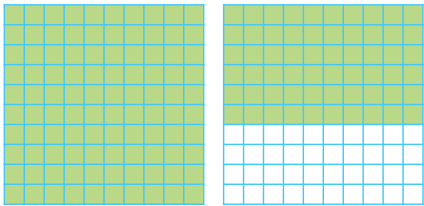
Example 3: Convert Percents to Fractions and Decimals

Convert each percent to a decimal and a fraction.

- a) 160% b) 0.35% c) $25\frac{3}{5}\%$

Solution

- a) You could represent 160% using hundred grids.



$$\frac{100}{100} + \frac{60}{100} = \frac{160}{100}$$

$\frac{160}{100}$ is equivalent to $\frac{16}{10}$ or $\frac{8}{5}$.

You can interpret $\frac{160}{100}$ as $160 \div 100 = 1.6$.

So, 160% can be expressed as 1.6, $\frac{16}{10}$, or $\frac{8}{5}$.

How do you know $\frac{160}{100}$ and $\frac{8}{5}$ are equivalent?

Is 160% greater than or less than one whole?

- b) Percent means out of 100. So, 0.35% can be written as $\frac{0.35}{100}$.

You can interpret $\frac{0.35}{100}$ as $0.35 \div 100 = 0.0035$.

$0.0035 = \frac{35}{10\,000}$, since the 5 is in the ten thousandths place.

$\frac{35}{10\,000}$ can be written in lowest terms as $\frac{7}{2000}$.

What factors of 35 divide evenly into 10 000?

Is 0.35% greater than or less than $\frac{1}{100}$?

- c) $25\frac{3}{5}\%$ can be expressed as $25\% + \frac{3}{5}\%$.

25% is 0.25 or $\frac{1}{4}$. You can interpret $\frac{3}{5}\%$ as $3 \div 5 = 0.6$.

$\frac{3}{5}\%$ would be $0.6 \div 100 = 0.006$.

$$\begin{aligned} \text{So, } 25\frac{3}{5}\% &= 0.25 + 0.006 \\ &= 0.256 \end{aligned}$$

That is the same as $\frac{256}{1000}$.

$25\frac{3}{5}\%$ can be expressed as 0.256 or $\frac{256}{1000}$.

What is $\frac{256}{1000}$ in lowest terms? Show your thinking.

Is $25\frac{3}{5}\%$ greater than or less than $\frac{1}{4}$?
Is $\frac{256}{1000}$ greater than or less than $\frac{1}{4}$? How do you know?

Show You Know

Convert each percent to a decimal and a fraction.

- a) 750% b) 0.3% c) $12\frac{3}{4}\%$

Example 4: Determine a Percent

For the past century, the north magnetic pole has been drifting across the Canadian Arctic. Prior to the 1970s, the magnetic pole was drifting at an average speed of 10 km/year. Since the 1970s, the speed at which the magnetic pole has been drifting has increased to about 50 km/year. The circumference of Earth is approximately 40 000 km.

- a) What percent is the current speed of the original speed?
- b) The circumference of Earth is approximately 40 000 km. At 50 km/year, what percent of Earth's circumference will the pole drift in one year?

Solution

- a) The current speed is 50 km/year.
The original speed is 10 km/year.
Divide to find what percent the current speed is of the original speed.
$$\frac{50}{10} = 5$$

Percent means out of 100. So, $5 = \frac{500}{100}$.

So, $5 = 500\%$

The current speed is 500% of the original speed.

- b) The circumference of Earth is 40 000 km.
The distance the pole drifts in one year is 50 km.
The amount of Earth's circumference travelled in one year is represented by

$$\begin{aligned}\frac{50}{40\,000} &= \frac{1}{800} \\ &= 0.00125\end{aligned}$$

$$0.00125 = 0.125\%$$

At 50 km/year, the pole will drift 0.125% or $\frac{1}{8}\%$ of Earth's circumference in one year.

0.125 is equivalent to the fraction $\frac{1}{8}$.

Show You Know

Suppose that the speed at which the pole is drifting increased to 75 km/year.

- a) What percent is 75 km/year of the original speed?
- b) At 75 km/year, what percent of 40 000 km would the pole drift in one year?

Key Ideas

- Fractions, decimals, and percents can be used to represent numbers in various situations.
- Percents can be written as fractions and as decimals.

$$\begin{array}{lll} \frac{1}{2}\% = 0.5\% & 150\% = \frac{150}{100} & 42\frac{3}{4}\% = 42.75\% \\ 0.5\% = \frac{0.5}{100} & = 1.5 \text{ or } 1\frac{1}{2} & 42.75\% = \frac{42.75}{100} \\ = 0.005 & & = 0.4275 \end{array}$$

Communicate the Ideas

1. Kaitlyn and Jordan are converting 0.003 to a percent. Who is correct? Show how you know.

Kaitlyn:
 $0.003 = 3\%$

Jordan:
 $0.003 = 0.3\%$

2. Which number does not have the same value as the other three? Explain your reasoning.

$$\frac{12}{5} \quad 2.4 \quad 250\% \quad \frac{60}{25}$$

3. Teammates Mark and Jonas are discussing the outcome of a game. Mark says their team scored 500% as many goals as the other team and Jonas says they scored five times as many goals as the other team. Can they both be correct? Explain how you know.

Check Your Understanding

Practise

For help with #4 and #5, refer to Example 1 on page 131.

4. Convert each fraction to a decimal and a percent.
- a) $\frac{1}{250}$ b) $\frac{81}{200}$ c) $\frac{7}{5}$

5. Rewrite each fraction as a decimal and a percent.

a) $\frac{51}{30}$ b) $\frac{21}{200}$ c) $\frac{3}{500}$

For help with #6 and #7, refer to Example 2 on page 132.

6. Convert each decimal to a percent and a fraction.
- a) 0.0072 b) 0.548 c) 3.45

7. Change each decimal to a percent and a fraction.

- a) 0.256 b) 0.0005 c) 6.5

For help with #8 and #9, refer to Example 3 on page 133.

8. Convert each percent to a decimal and a fraction.

- a) 248% b) 0.56% c) $75\frac{3}{4}\%$

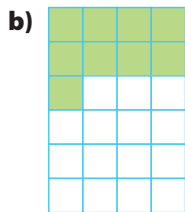
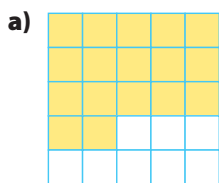
9. Express each percent as a decimal and a fraction.

- a) $5\frac{9}{10}\%$ b) 550% c) 0.8 %

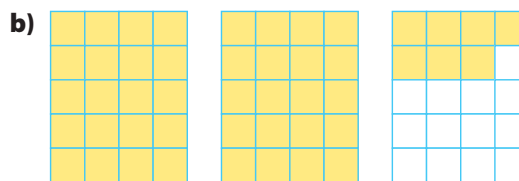
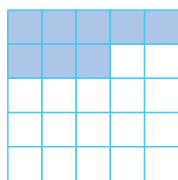
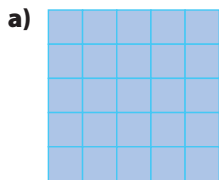
10. Copy and complete the following table. The first row is completed for you.

Percent	Fraction	Decimal
165%	$\frac{165}{100}$	1.65
a) 230%		
b) 0.38%		
c) 19.9%		

11. Express the shaded portion of each diagram as a fraction, a decimal, and a percent.



12. If one completely shaded grid represents one whole, express the shaded portion of each diagram as a fraction, a decimal, and a percent.



Apply

For help with #13 and #14, refer to Example 4 on page 134.

13. Several years ago Claire bought the first issue of a popular comic book for \$10. At a recent auction, it sold for \$200. What percent is the new value of the comic book of the price several years ago?
14. A snack contains 0.9 g of fat. Suppose that in one day, Shaun consumed a total of 40 g of fat, including the snack. What percent of Shaun's total fat consumption does the snack represent? What is this percent as a decimal and as a fraction?
15. Use hundred grids to help place the following numbers in ascending order. 145%, $\frac{5}{8}\%$, 1.32, 0.65, 33.5%, 0.6%
16. A miner found 12 g of gold in a 2700-g sample of ore. What percent of the sample is gold, to the nearest tenth of a percent? What is the percent as a repeating decimal and as a fraction in lowest terms?

Literacy Link

A repeating decimal contains a digit or group of digits that repeat forever. You can write a repeating decimal using bar notation.

$$0.333\ 33\dots = 0.\overline{3} \quad 0.454\ 545\dots = 0.\overline{45}$$

17. A fundraising coordinator is preparing an advertising flyer for an upcoming event. She wants to use either a fraction or a decimal number to represent each of the percents in the following statements. Decide whether a fraction or a decimal number is better and rewrite each statement using your chosen representation. Justify your choices.

- a) Ticket sales are 130% of what they were at this time last year.
- b) We are already at $60\frac{1}{2}\%$ of our target and we just started!
- c) We have managed to cut our costs by 0.75%.

18. A fisheries worker recorded the following species and numbers of fish passing by a fish counter. Copy and complete the following table.

Species	Number	Percent of Total	Fraction of Total	Decimal Equivalent
Chinook	143			
Coho	122			
Steelhead	2			

19. Over five years, the circulation of a magazine increased from 25 000 copies to 150 000 copies. What percent is the new circulation of the circulation five years ago? What is this percent as a decimal and as a fraction?

20. Kim's resting heart rate was 75 beats per minute. A trainer advised Kim to have a portion of her workout at 90 beats per minute and a portion at 125 beats per minute, but not to exceed 150 beats per minute. Express each heart rate compared to the resting heart rate as a percent, a fraction, and a decimal.



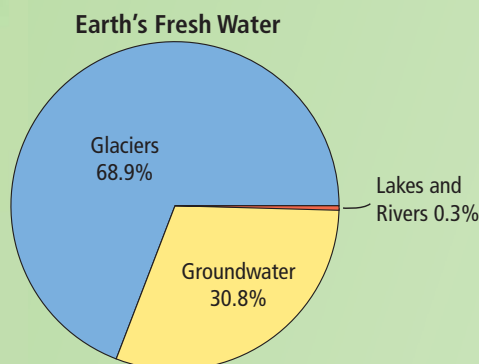
Extend

21. Copy and complete the first three rows of the table. Use the patterns in the first three rows to complete the last two rows.

Percent	Decimal	Fraction
a) 1000		
b)	5.00	
c)		$\frac{5}{2}$
d)		
e)		

MATH LINK

Represent the percents shown in the circle graph in two other ways.



Did You Know?

In 2002, NASA launched two satellites to measure groundwater amounts from space! These satellites use gravity to *weigh* Earth's groundwater.

4.3

Percent of a Number

Focus on...

After this lesson, you will be able to...

- solve problems that involve percents less than 1%
- solve problems involving percents greater than 100%
- solve problems involving fractional percents



You often use percents to make comparisons and help make decisions. A fundraising team is raising money for a relief organization. The team wants to use their profits for several purposes.

How could the team use percents to decide how much money to donate for each purpose?

Literacy Link

Profit is the amount of money left over after all expenses are paid.

Explore the Math

How can you solve problems involving percents?

Last year the fundraising team ran a school store and made $50\frac{1}{2}\%$ profit. The school store usually has total sales of about \$10 000 per year.

1.
 - a) How much is 50% profit?
 - b) How much is 1% profit?
 - c) How much is $\frac{1}{2}\%$ profit?
 - d) How much is $50\frac{1}{2}\%$ profit?
2. The committee wants to donate 10% of the store profits for providing food.
 - a) What is 10% of the profit calculated in #1d)?
 - b) How could you determine 10% of a number mentally? Explain.

3. The committee knows that access to clean drinking water is critical in preventing serious illness. They would like to donate 20% of their profits for providing clean drinking water. How could you determine 20% of the profits mentally using your answer to #2?
4. Oral rehydration therapy (ORT) is a simple yet inexpensive medicine designed to fight dehydration.
 - a) If it costs \$0.10 to prepare 1 L of ORT solution, how many litres of ORT can be prepared using the money from the $\frac{1}{2}\%$ portion of the store profits?
 - b) If the average adult needs about 4 L of ORT for adequate rehydration, how many adults can be treated using the $\frac{1}{2}\%$ profit?

Did You Know?

Oral rehydration therapy (ORT) is a mixture of water, salt, and sugar. It is used to restore necessary water content to people who have become dehydrated because of illness or a lack of proper drinking water. What do you think is the purpose of the salt and the sugar?

Reflect on Your Findings

5. How can you use mental math techniques to help you find the percent of a number?

Example 1: Use Mental Math to Find the Percent of a Number

Use mental math to determine each of the following.

- a) 150% of \$5 b) 0.1% of \$1000 c) $1\frac{1}{2}\%$ of \$20 000

Solution

- a) 150% is $100\% + 50\%$.
 100% of 5 is 5.
 50% is half of 100%.

Use halving to find 50% of 5.
 Half of 5 is 2.5.

$$150\% \text{ of } 5 \text{ is } 5 + 2.5.$$

$$5 + 2.5 = 7.5$$

So, 150% of \$5 is \$7.50.

- b) To determine 0.1% of \$1000, divide repeatedly by tens.
 100% of 1000 is 1000.
 10% of 1000 is 100.
 1% of 1000 is 10.
 0.1% of 1000 is 1.

So, 0.1% of \$1000 is \$1.

Literacy Link

Halve means divide by two.
Double means multiply by two.

Strategies

Look for a Pattern

You could also determine 1.5% of \$20 000 as:
 30% of 20 000 is 6000.
 3% of 20 000 is 600.
 1.5% of 20 000 is 300.



c) Divide repeatedly by tens to reach 1%, and then divide by two.

100% of 20 000 is 20 000.

10% of 20 000 is 2000.

1% of 20 000 is 200.

$\frac{1}{2}$ % of 20 000 is $200 \div 2$.

$200 \div 2 = 100$

$1\frac{1}{2}$ % of 20 000 is $200 + 100$.

$200 + 100 = 300$

So, $1\frac{1}{2}$ % of \$20 000 is \$300.

Show You Know

Use mental math to determine each of the following.

- a) 350% of \$10
- b) 0.1% of \$5000
- c) $2\frac{1}{10}$ % of \$20 000

Example 2: Calculate the Percent of a Number

- a) A survey showed that $\frac{1}{4}$ % of 800 students use inline skates to get to school. How many of the 800 students in a school use inline skates to get to school?
- b) $30\frac{3}{4}$ % of 400 students surveyed said they own a cell phone. How many of the students own a cell phone?
- c) Adele invested \$40.12 in a savings plan at the beginning of the year. By the end of the year her investment was worth 120% of its original value. How much was her investment worth, to the nearest cent?

Literacy Link

In math, the word *of* often means to multiply.

10% of 800 is 80.

1% of 800 is 8.

$\frac{1}{4}$ % of 800 is 2.



Solution

a) Determine $\frac{1}{4}$ % of 800.

$$\frac{1}{4}\% = 0.25\%$$

Divide by 100 to write the percent as a decimal.

$$0.25 \div 100 = 0.0025$$

$$0.0025 \text{ of } 800 = 0.0025 \times 800 = 2$$

So, two students out of 800 students used inline skates to get to school.

b) Determine $30\frac{3}{4}\%$ of 400.

Since $\frac{3}{4}\%$ is 0.75%, $30\frac{3}{4}\% = 30.75\%$.

Divide by 100 to write the percent as a decimal.

$$30.75 \div 100 = 0.3075$$

$$0.3075 \text{ of } 400 = 0.3075 \times 400 = 123$$

So, 123 of the 400 students own a cell phone.

10% of 400 is 40. **M E**
30% of 400 is 120.
1% of 400 is 4.
 $\frac{1}{4}\%$ of 400 is 1.
 $\frac{3}{4}\%$ of 400 is 3.
 $30\frac{3}{4}\%$ of 400 is $120 + 3$.
 $120 + 3 = 123$

c) Determine 120% of \$40.12.

Divide by 100 to write the percent as a decimal.

$$120 \div 100 = 1.2$$

$$1.2 \text{ of } 40.12 = 1.2 \times 40.12 \approx 48.144$$

So, 120% of \$40.12 is \$48.14.

To the nearest cent means to the nearest hundredth of a dollar.

100% of 40.12 is 40.12. **M E**
10% of 40.12 is 4.012.
20% of 40.12 is 8.024.
120% of 40.12 is $40.12 + 8.024$.
 $40.12 + 8.024 = 48.144$

Show You Know

Determine the percent of each number.

a) 160% of \$53.27

b) $\frac{3}{4}\%$ of 135

c) $55\frac{8}{10}\%$ of 500

Key Ideas

- You can use mental math strategies such as halving, doubling, and dividing by ten to find the percents of some numbers.
- To calculate the percent of a number, write the percent as a decimal and then multiply by the number.

$$12\frac{1}{2}\% \text{ of } 50 = 0.125 \times 50 = 6.25$$

$$12\frac{1}{2}\% = 12.5\% = 0.125$$

Communicate the Ideas

1. Explain to a classmate how you could use mental math to find each of the following.

a) 300% of 40

b) 0.5% of 120

c) $10\frac{1}{2}\%$ of 80

2. Describe two ways to find 6% of 120.

Check Your Understanding

Practise

For help with #3 and #4, refer to Example 1 on pages 139–140.

3. Use mental math to determine each of the following.
 - a) 300% of 2000
 - b) $1\frac{1}{4}\%$ of 60
 - c) 0.1% of 40
4. Use mental math to find the following.
 - a) 20% of 60
 - b) 250% of 400
 - c) $10\frac{1}{2}\%$ of 100

For help with #5 and #6, refer to Example 2 on pages 140–141.

5. Determine the percent of each number. Give your answer to the nearest hundredth.
 - a) $\frac{2}{5}\%$ of 325
 - b) $15\frac{1}{4}\%$ of 950
 - c) 175% of \$125.50
6. What is the percent of each number? Give your answer to the nearest hundredth.
 - a) $\frac{5}{8}\%$ of 520
 - b) $75\frac{2}{5}\%$ of 200
 - c) 250% of \$76.50

Apply

7. Two hundred tickets are being sold for a school draw.
 - a) What is your chance of winning with one ticket? Express your answer as a percent.
 - b) How many tickets would you need to purchase to have a 2.5% chance of winning?
8. The original price of a jacket was \$84.00. A store manager marked the price down by $25\frac{1}{2}\%$. By how much was the price reduced?
9. The highest point in Canada is Mount Logan, which is in the Yukon Territory. Mount Logan is 159% as high as the highest point in Alberta, Mount Columbia. The elevation of Mount Columbia is 3747 m. What is the elevation of Mount Logan?



10. When water freezes, its volume increases by approximately 10%.
 - a) By how much does the volume of 750 mL of water increase when it freezes?
 - b) What is the volume of ice created?

11. The area of Canada is approximately 9 984 670 km². The area of Manitoba is about $6\frac{1}{2}\%$ of the area of Canada. What is the area of Manitoba to the nearest square kilometre?

12. A manufacturer of electric hybrid vehicles claims its vehicle will travel 200% as far as its regular vehicle on a full tank of gas. If the regular vehicle travels an average of 550 km on a full tank, how far will the hybrid go?



13. Suppose a real estate agent receives 5% commission on the first \$200 000 of a house's selling price, and 6% on the remaining amount.
- What does *commission* mean?
 - If a house sells for \$345 000, how much commission does the real estate agent make on the sale of the house?

Extend

14. 4% of 100 is the same as 8% of what number? Explain how you arrived at your answer.
15. A new video gaming system was auctioned on the Internet. The starting bid was \$100. The second bid was 135% of the first bid. The third bid was 257% of the second bid. There were then five more bids, each $10\frac{1}{2}\%$ over the previous bid. The winning bid came with only seconds left and was only 0.1% greater than the previous bid. What was the winning bid? What assumptions did you make to arrive at your answer?
16. Josephine scored 12 baskets out of 30 shots in her first basketball game this year. Her scoring average was then 40%. The next game, she made ten shots and raised her scoring average for both games to 50%. How many of the ten shots in the second game were baskets?

MATH LINK

Water conservation is very important to protect local fresh water supplies.

- Research at least three ways that your home, school, and community could reduce water consumption.
- Develop three water math problems that ask how much water you might save if you used some of these ways of conserving.

WWW Web Link

Did you know that a swimming pool cover can help reduce water loss by evaporation by 90%? To find data and tips on conserving water, go to www.mathlinks8.ca and follow the links.

4.4

Combining Percents

Focus on...

After this lesson, you will be able to...

- solve problems involving combined percents



Literacy Link

PST means provincial sales tax. PST varies by province.

GST means goods and services tax. GST is the same across Canada.

Jesse and Jenna have \$55 to purchase prizes for a school fundraiser. The items cost \$49.99 plus 5% GST and 7% PST. Do you think they will have enough money?

When they reach the cashier, they discover that the store has a one-day sale—they only have to pay 50% of the tax. How much tax do you think they will have to pay?

Explore the Math

How can you combine percents?

1. A store advertises 40% off. You purchase an item regularly priced at \$100.
 - a) What is the discount for the item?
 - b) What is the sale price of the item?
 - c) What percent of the original price are you paying?
 - d) How are the percent discount and the percent of the original price related? Use a grid to explain your answer.
 - e) How could you estimate the price of something that has a 40% discount?



2. Suppose GST is 5% and PST is 7%. You purchase an item for \$100.
 - a) Represent the GST and the PST on a hundred grid.
 - b) How much is the GST? the PST?
 - c) How much tax do you pay altogether?
 - d) What is your total tax as a percent of \$100? How does this percent value compare to the sum of the percent values for GST and PST?
 - e) What decimal could you multiply \$100 by to find the total cost including tax?

3. Suppose an item regularly priced at \$200 is on sale for 10% off. PST is 7% and GST is 5%.
 - a) Write a multiplication expression to show how to determine the price of the item with the 10% discount applied.
 - b) Write a multiplication expression to show how to determine the total amount of tax on the item in part a).
 - c) What is the total cost of the item including tax?

4. Caroline purchased a sweatshirt originally priced at \$50. It was on sale for 25% off. The PST where she lives is 5%. The GST is 5%.
 - a) What is the cost of the sweatshirt before tax?
 - b) Caroline used the single expression 10% of 75% of \$50 to determine the total amount of tax. Explain why Caroline's expression is correct.

WWW Web Link

Not all provinces have the same PST. To learn more about PST rates, go to www.mathlinks8.ca and follow the links. What is the rate of PST where you live?

Did You Know?

In Saskatchewan, PST is 5%. In Alberta there is no PST. The city of Lloydminster, Saskatchewan, is half in Alberta!
A provincial law states that no PST is paid in the whole city. What might be a reason for the law?

Reflect on Your Findings

5. a) Describe two ways that you can calculate the total tax on an item.
- b) Which method do you prefer to use? Explain why.

Example 1: Combined Percents

Suppose GST is 5% and PST is 7%. Calculate the total tax and total cost of a sound system that is priced at \$250.

Solution

Method 1: Calculate the Taxes Separately

The GST is 5%.

5% is 0.05.

Multiply by the price to determine the amount of GST.

$$0.05 \times 250 = 12.5$$

The amount of GST is \$12.50.

10% of 250 is 25. **M E**
 5% of 250 is 12.5.
 1% of 250 is 2.5.
 7% of 250 is
 $12.5 + 2.5 + 2.5 = 17.5$.

The PST is 7%.

7% is 0.07.

Multiply by the price to determine the amount of PST.

$$0.07 \times 250 = 17.5$$

The amount of PST is \$17.50.

Add the two tax amounts.

$$12.50 + 17.50 = 30.00.$$

The total tax is \$30.00.

$$\begin{aligned} \text{Total Cost} &= \text{Cost of Item} + \text{Total Tax} \\ &= 250.00 + 30.00 \\ &= 280.00 \end{aligned}$$

The total cost of the sound system is \$280.00.

Literacy Link

You can combine percents by adding individual percent values together.

Method 2: Combine the Tax Percents First

The GST is 5%. The PST is 7%.

The combined tax is 5% + 7% or 12%.

Convert the percent to a decimal.

$$12\% = 0.12$$

Multiply by the price to determine the total amount of tax.

$$0.12 \times 250 = 30$$

The total tax is \$30.00.

$$\begin{aligned} \text{Total Cost} &= \text{Cost of Item} + \text{Total Tax} \\ &= 250.00 + 30.00 \\ &= 280.00 \end{aligned}$$

The total cost of the sound system is \$280.00.

Method 3: Combine the Cost and Tax Percents

You could use a percent greater than 100% to find the total cost.

The cost of the sound system is 100%.

The PST is 7%.

The GST is 5%.

The cost of the sound system expressed as a percent of the original cost is 100% + 7% + 5% or 112%.

Convert the percent to a decimal.

$$112\% = 1.12$$

Multiply by the price to determine the total cost.

$$1.12 \times 250 = 280$$

The total cost of the sound system is \$280.00.

Show You Know

A backpack costs \$35. Use the method of your choice to find the total cost of the backpack if GST is 5% and PST is 6%. Use another method to check your work.

Example 2: Percent of a Percent

Sports R Us offers a 10% off discount one day and then an additional 10% off the sale price the next day. Sports Galore offers a 20% discount on one day only. Keifer wants to buy a new goalie mask that has a regular price of \$200 at both stores.

Sports Galore
20% off one day only!

Sports R Us
10% off already reduced prices!

- Which store gives the better buy? Explain your reasoning.
- What single percent discount is equivalent to a discount of 10% one day followed by an additional discount of 10% off the sale price the second day?

Solution

a) Sports R Us

The discount on the first day is 10% of \$200.
 $10\% \text{ of } 200 = 0.10 \times 200$
 $= 20$

Subtract to find the discount price.

$$200 - 20 = 180$$

The discount price on the first day is \$180.

The discount on the sale price the second day is 10% of \$180.

$$10\% \text{ of } 180 = 0.10 \times 180$$
$$= 18$$

Subtract to find the discount price.

$$180 - 18 = 162$$

The discount price after the second day is \$162.

Sports Galore

The discount is 20% of \$200.
 $20\% \text{ of } 200 = 0.20 \times 200$
 $= 40$

Subtract to find the discount price.

$$200 - 40 = 160$$

The discount price is \$160.

Sports Galore gives a better buy than Sports R Us. The 10% discount followed by another 10% discount is not the same as a 20% discount because the discount on the second day is only 10% of \$180 and not 10% of \$200.

b) The original price is \$200.

The selling price after two 10% discounts at Sports R Us is \$162.

Subtract to find the total amount of the discount.

$$200 - 162 = 38$$

The total amount of the discount is \$38.

Determine what percent the total discount is of the original price.

$$\frac{38}{200} = 0.19$$

The total discount is 19% of the original price.

A 19% discount is less than the single discount of 20% offered by Sports Galore.

Show You Know

What is the final sale price at each store? Which is a better buy?

Explain your thinking.

Store A: 50% off one day only

Store B: 25% off one day followed by 25% off the reduced price the second day

Key Ideas

- Percents can be combined by adding to solve problems. $5\% + 7\% = 12\%$
- To calculate the increase in a number,
 - You can add the combined percent amount to the original number.
 12% of $100 = 0.12 \times 100 = 12$
 $100 + 12 = 112$
 - You can multiply the original number by a single percent greater than 100.
 112% of $100 = 1.12 \times 100 = 112$
- Percents of percents can be used to determine amounts that result from consecutive percent increases or decreases.

Communicate the Ideas

1. Draw a diagram to show how you could represent the cost of a \$100 item with and without tax.
2. Your friend shows you how to calculate the cost of an item including tax using several steps. You tell her that you can do the calculation in one step. Show how you would do this.
3. Kyle says that a population increase of 15% one year followed by an increase of 10% the next year is the same as a population increase of 25% over two years. Is Kyle correct? Explain your reasoning.

Check Your Understanding

Practise

For help with #4 and #5, refer to Example 1 on pages 145–146.

4. Chris purchased the following items:
 - 2 binders at \$4.99 each
 - 1 math set for \$3.99
 - a backpack for \$19.99

Find the total cost including 5% GST and 7% PST.

5. Ravi purchased 3 DVDs for \$19.99 each. Find the total cost for the DVDs including 5% GST and 6% PST.

For help with #6 and #7, refer to Example 2 on page 147.

6. A store discounted items by 50% off the original price one week. The following week an additional 10% was taken off the already reduced price. The regular price of a CD player was \$85.00. What is the reduced price in the second week?

7. A herd of 100 caribou was moved to a new location. The population increased by 10% the first year and then increased by 20% the second year.
- Find the population after the second year.
 - Explain why there was not a 30% increase in population over the two years.

8. Copy and complete the following table. Use 5% GST and the percent of PST applicable to where you live.

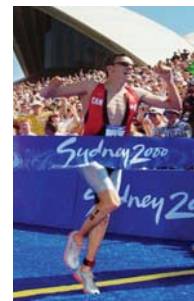
Item Purchased	Price	Total Tax	Total Cost
a) Boots	\$119.99		
b) Pants	\$89.99		
c) Gloves	\$39.99		
d) Helmet	\$189.99		

Apply

9. Arjay was thinking of buying a car worth \$23 000, but delayed purchasing the car for a year. During that year, the cost of the car increased by 3.2%.
- What was the price of the car when Arjay purchased it?
 - What was the total cost of the car including 5% GST and 5% PST?
10. What is the total cost for four tires that sell for \$85 each, plus 5% GST and a 1.5% environment tax?

11. A student is awarded a \$1000 scholarship and places it in an account that pays 3% simple interest per year.
- What is the total value of the scholarship amount at the end of the second year?
 - What is the single percent increase in value of the scholarship after two years?

12. Simon Whitfield of Victoria, British Columbia, won the men's triathlon at the Sydney Olympics. The race consisted of a 1.5-km swim in Sydney Harbour, a 40-km bike ride through Sydney and a 10-km run.



- What percent of the race distance is each component? Express your answer to the nearest tenth of a percent.
- What percent of the race distance is spent on land? Express your answer to the nearest tenth of a percent.

Extend

13. A ski jacket has been marked down on three occasions, first 20% off, then 25% off the new price, and finally 50% off the previous price. What is the overall percent saved?
14. The selling price of a DVD player is 35% more than its cost. It is sold at a discount of 20% off the selling price. How much does the store still gain?

MATH LINK

- In one day, a dripping faucet wastes about 25 L of water. A regular toilet flush uses 6 L of water per flush. If you flush your toilet 30 times a day, what percent of the water used by your toilet is wasted by the dripping faucet?
- $\frac{3}{10}$ % of the world's fresh water is held in rivers and lakes. Approximately 9% of that water is used for industry and may be returned to the environment polluted. What percent of the world's fresh water is used by industry?