3.1 Squares and Square Roots

Focus on...

After this lesson, you will be able to...

- ❑ determine the square of a whole number
- ❑ determine the square root of a perfect square

Literacy Link

A *square number* is the product of the same two numbers. 3 x 3 = 9, so 9 is a square number.

A square number is also known as a *perfect square*. A number that is not a perfect square is called a *non-perfect square*.

The Pythagoreans were members of an academy of study that existed 2500 years ago. They created square numbers by arranging pebbles in equal numbers of rows and columns. Nine pebbles could be arranged in three rows and three columns. Nine is a square number because $3 \times 3 = 9$. The picture shows the first four square numbers that the Pythagoreans found: 1, 4, 9, and 16. How can you determine the next square number?

Did You Know?

Pythagoras (about 580–500 B.C.E.) was the leader of a group of academics called the Pythagoreans. They believed that patterns in whole numbers could help explain the universe.

Explore the Math

Materials

- square tiles

How can you identify a perfect square?

1. Use square tiles to make five rectangles with the dimensions shown.

   What is the area of each rectangle?

Length (cm)	Width (cm)
5	3
8	2
9	1
4	3
9	4

2. Try to rearrange the tiles in each rectangle to make a square.
   a) Which rectangles can you make into squares?
   b) What is the side length of each square?
   c) How is the area of each square related to its side length?

3. a) Choose three perfect squares and three non-perfect squares.
   b) Express each number as a product of prime factors.
   c) For each number, how many times does each prime factor appear? Compare your results with a partner's results.

4. a) What do all of the perfect squares have in common?
   b) What do all of the non-perfect squares have in common?

> **Literacy Link**
>
> **Prime Numbers and Prime Factors**
>
> A *prime number* is a whole number greater than 1 that has only two factors: 1 and itself.
>
> *Prime factors* are factors that are prime numbers.
>
> For example, the prime factors of 10 are 2 and 5.

Reflect on Your Findings

5. a) How can square tiles help you to determine if a number is a perfect square?
   b) How can prime factors help you to determine if a number is a perfect square?

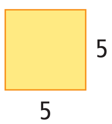
Example 1: Identify Perfect Squares

prime factorization

- a number written as the product of its prime factors
- the prime factorization of 6 is 2 × 3

perfect square

- a number that is the product of the same two factors
- has only an even number of prime factors
- 5 × 5 = 25, so 25 is a perfect square



a) Determine the **prime factorization** of the following numbers: 24, 36, 81.

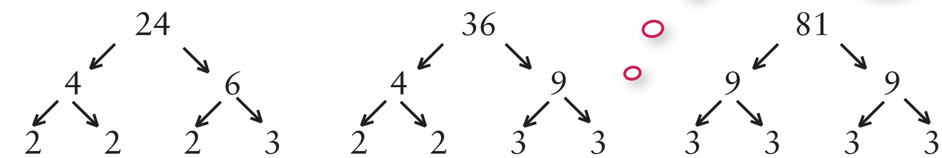
b) Which of the numbers is a **perfect square**? Explain.

c) For each number that is a perfect square, draw the square and label its side length.

Different factor trees are possible to arrive at the same prime factorization.

Solution

a)



$24 = 2 \times 2 \times 2 \times 3$ $36 = 2 \times 2 \times 3 \times 3$ $81 = 3 \times 3 \times 3 \times 3$

b) To be a perfect square, each prime factor in the prime factorization must occur an even number of times. 36 and 81 are perfect squares because each prime factor occurs an even number of times.

$36 = 2 \times 2 \times 3 \times 3$ two factors of 2, two factors of 3

$81 = 3 \times 3 \times 3 \times 3$ four factors of 3

24 is not a perfect square because at least one of the prime factors occurs an odd number of times.

$24 = 2 \times 2 \times 2 \times 3$ three factors of 2, one factor of 3

c) To determine the side length of the squares, look at the product of prime factors for the area.

$36 = 2 \times 2 \times 3 \times 3$ $81 = 3 \times 3 \times 3 \times 3$

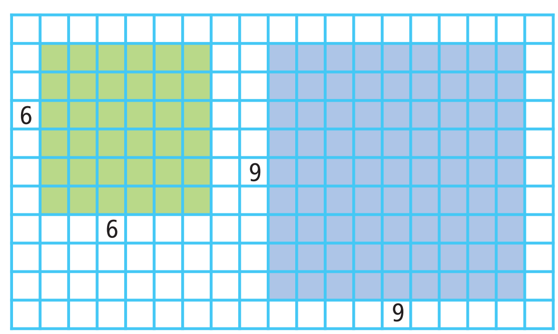
Rearrange the prime factors into two equal groups.

$36 = (2 \times 3) \times (2 \times 3)$

$36 = 6 \times 6$

$81 = (3 \times 3) \times (3 \times 3)$

$81 = 9 \times 9$



Show You Know

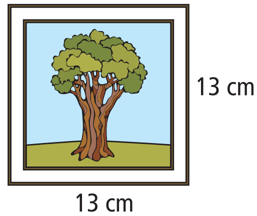
Write the prime factorization of each number. Which number is not a perfect square? Explain how you know.

a) 45 **b)** 100

Example 2: Determine the Square of a Number

Determine the area of a square picture with a side length of 13 cm.

Solution



$A = s^2$
$A = 13^2$
$A = 13 \times 13$
$A = 169$
The area is 169 cm^2.

area of a square = side length × side length
$A = s \times s$
$A = s^2$

Strategies
Draw a Diagram

Literacy Link

You can write a repeated multiplication like 13×13 as a square: $13 \times 13 = 13^2$.

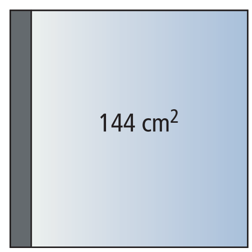
13^2 is read as thirteen squared.

Show You Know

Determine the area of a square with a side length of 16 mm.

Example 3: Determine the Square Root of a Perfect Square

Edgar knows that the square case for his computer game has an area of 144 cm^2. What is the side length of the case?



Solution

Method 1: Use Inspection

To find the side length, determine what positive number when multiplied by itself equals 144.

$12 \times 12 = 144$

The **square root** of 144 is 12, or $\sqrt{144} = 12$.

The side length is 12 cm.

square root

- a number that when multiplied by itself equals a given value
- 6 is the square root of 36 because $6 \times 6 = 36$

Method 2: Use Guess and Check

Find the positive value for the blank boxes.

■ × ■ = 144

$10 \times 10 = 100$ Too low
$13 \times 13 = 169$ Too high
$12 \times 12 = 144$ Correct!
$12 = \sqrt{144}$

The side length is 12 cm.

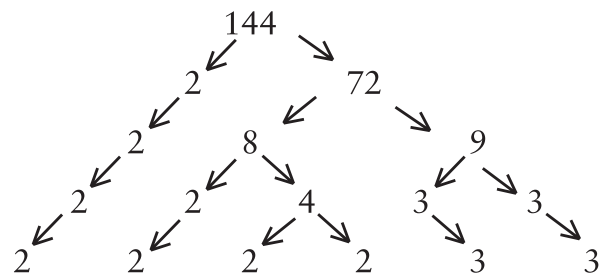
Literacy Link

Reading Square Roots

The symbol for square root is $\sqrt{\ }$.

Read $\sqrt{9}$ as the square root of 9, square root 9, or root 9.

Method 3: Use Prime Factorization



The prime factorization of 144 is $2 \times 2 \times 2 \times 2 \times 3 \times 3$.

Rearrange the prime factors into two equal groups.

$144 = (2 \times 2 \times 3) \times (2 \times 2 \times 3)$

$144 = 12 \times 12$

$\sqrt{144} = 12$

The side length is 12 cm.

Tech Link

You can use a calculator to find the square root of a number. Try the following key sequences on your calculator. Then, record the one that works on your calculator.

C 144 √ =

or

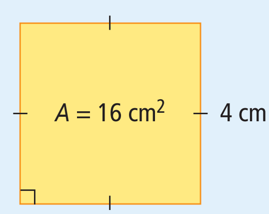
C √ 144 =

Show You Know

Determine the side length of a square with an area of 196 cm^2.

Key Ideas

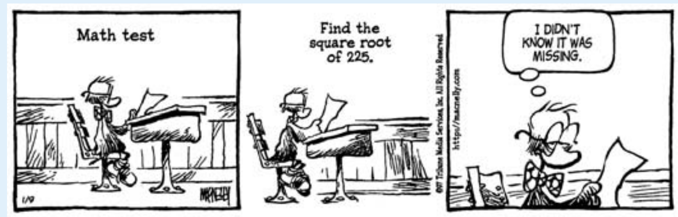
- The square of a number is the number multiplied by itself.
  $5 \times 5 = 25$, or $5^2 = 25$
- The square of a whole number is a perfect square. $2^2 = 4$
  So, 4 is a perfect square.
- The square of a number can be thought of as the area of a square.
  $4^2 = 16$
  The area is 16 cm^2.



- The square root of a number can be thought of as the side length of a square.
  $\sqrt{16} = 4$
  The side length is 4 cm.
- The square root of a value is a number that when multiplied by itself equals the value.
  $6 \times 6 = 36$, so $\sqrt{36} = 6$
- In the prime factorization of a perfect square, there is an even number of each prime factor.
  $36 = 2 \times 2 \times 3 \times 3$ two factors of 2, two factors of 3

Communicate the Ideas

1. Explain how to square the number 7.



2. How would you use prime factorization to determine the square root of 225? Compare your answer with a classmate's.

3. The factors of 36 are 1, 2, 3, 4, 6, 9, 12, 18, and 36. Use words and/or diagrams to explain how you know which factor is the square root of 36.

4. Explain how squaring a number is the reverse of finding the square root of a number. Include an example with your explanation.

Check Your Understanding

Practise

For help with #5 to #8, refer to Example 1 on page 82.

5. a) Determine the prime factorization of 4.
   b) Is 4 a perfect square? Explain.
   c) Draw the square and label its side length.

6. A rectangle has an area of 64 m^2.
   a) Determine the prime factorization of 64.
   b) Is 64 a perfect square? Explain.
   c) Draw a square with that area and label its side length.

7. Write the prime factorization of each number. Identify the perfect squares.
   a) 42 b) 169 c) 256

8. Determine the prime factorization of each number. Which numbers are perfect squares?
   a) 144 b) 60 c) 40

For help with #9 to #12, refer to Example 2 on page 83.

9. What is the area of a square with each side length?
   a) 10 b) 16

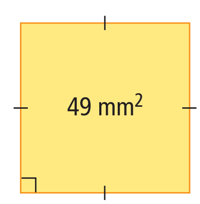
10. Determine the area of a square with each side length.
    a) 20 b) 17

11. What is the square of each number?
    a) 9 b) 11

12. Determine the square of each number.
    a) 3 b) 18

For help with #13 to #16, refer to Example 3 on pages 83–84.

13. What is the side length of the square shown?



14. Determine the side length of a square with an area of 900 cm^2.

15. Evaluate.

a) $\sqrt{49}$ **b)** $\sqrt{64}$ **c)** $\sqrt{625}$

16. Determine the value.

a) $\sqrt{9}$ **b)** $\sqrt{25}$ **c)** $\sqrt{1600}$

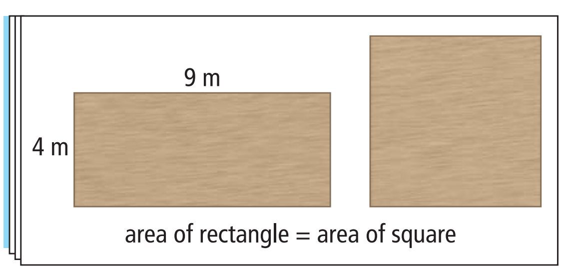
Apply

17. A fridge magnet has an area of 54 mm^2. Is 54 a perfect square? Use prime factorization to find the answer.

18. A floor mat for gymnastics is a square with a side length of 14 m. What is the area of the floor mat in square metres?

19. The gym teacher told the students to run twice around the perimeter of the school field. The area of the square field is 28 900 m^2. What distance did the students run?

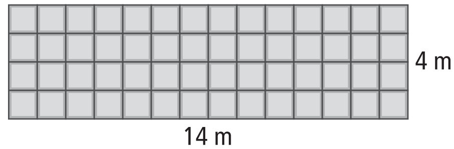
20. Adam's uncle has instructions for building a shed. One page of the instructions, shown below, is not very clear.



a) What is the area of the rectangle?

b) What is the side length of the square?

21. Kate is going to put a patio in her backyard. The patio stones she is using each have an area of 1 m^2. She has created the rectangular design shown.



a) What is the area of the patio?

b) What are the dimensions of another rectangular patio she could build with the same area?

c) Kate decides to make a patio with the same area but she wants it to be a square with whole number side lengths. Is this possible? Explain your reasoning.

22. The world's largest city square is Tiananmen Square in Beijing, China. It has an area of 396 900 m^2.

a) What are the dimensions of the square?

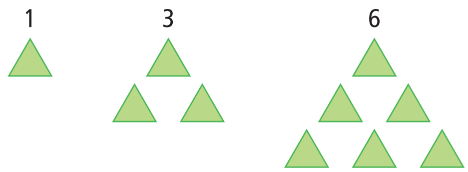
b) If the square had dimensions of 629 m by 629 m, what would be the area?

c) If the square had an area less than 394 000 m^2 and greater than 386 000 m^2, what are all of the possible whole number dimensions that it could have?

23. A helicopter landing pad has a square shape. The area is 400 m^2. Use prime factorization to find the side length of the pad.

Extend

24. The first three triangular numbers are



a) What are the next three triangular numbers?

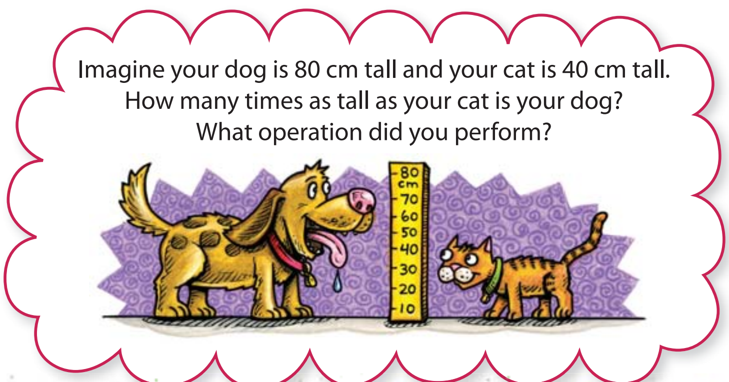
b) Add together any two consecutive triangular numbers. What do you notice about the sums?

25. A square digital photo on the computer has an area of 144 cm^2.

a) What is the side length of the photo?

b) The photo is enlarged so that the side length is now 36 cm. What is the area of the enlarged photo?

c) How many times as large as the original area is the enlarged area?



d) How many times as large as the original side length is the enlarged side length?

e) Use what you know about the square root of a perfect square to identify the relationship between the numbers in parts c) and d).

26. a) Determine which of the following numbers are perfect squares: 10, 100, 1000, 10 000, 100 000.

b) State the square root of each perfect square.

c) Choose one of the numbers that is not a perfect square. Explain how you know that it is not a perfect square.

d) Describe a quick method for determining mentally if the numbers are perfect squares.

e) Use your method in part d) to decide if 1 000 000 000 is a perfect square.

27. a) Determine the square root of each number: 6400, 640 000, 64 000 000.

b) Describe a quick method for determining mentally the square root of each number in part a).

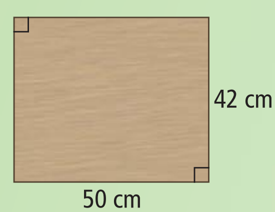
c) Explain why this method does not work for evaluating $\sqrt{640}$.

d) Use your method in part b) to evaluate $\sqrt{640\,000\,000\,000}$. Explain how you determined the answer.

MATH LINK

Chess is played on a square board. The board is made up of 32 white squares and 32 dark squares.

You decide to make your own chessboard. You are going to cut the board out of the 42 cm x 50 cm piece of wood shown.



Each square on the board will have whole number side lengths. The chess pieces fit on squares that are no smaller than 9 cm^2. What are all of the possible dimensions that your board could have?

3.2 Exploring the Pythagorean Relationship

Focus on...

After this lesson, you will be able to...

- ❑ model the Pythagorean relationship
- ❑ describe how the Pythagorean relationship applies to right triangles

Right triangles are found in art, construction, and many other objects. The sail for this sailboat is a right triangle. What makes this shape so special? You will explore some important properties of right triangles in this lesson.

Explore the Math

Materials

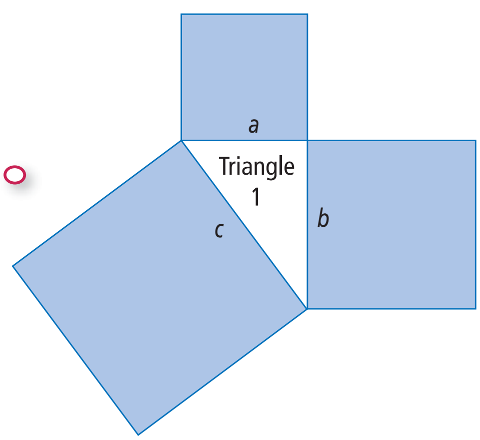
- centimetre grid paper
- scissors
- transparent tape
- protractor

What is a relationship that applies to right triangles?

1. From a piece of centimetre grid paper, cut out three squares with the following dimensions:
6 cm × 6 cm   8 cm × 8 cm   10 cm × 10 cm

2. Arrange the squares to form Triangle 1 as shown. Tape the squares onto a sheet of paper. Label Triangle 1.

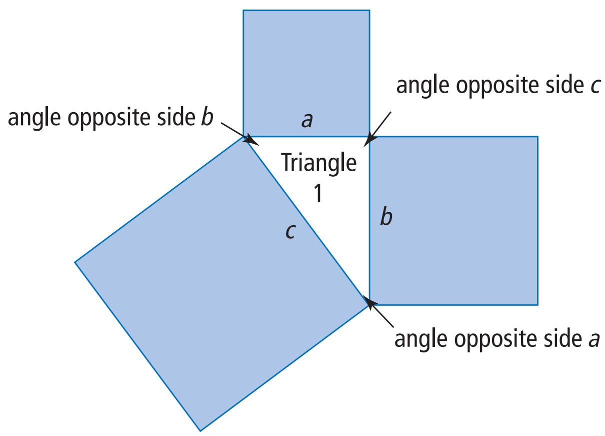
The length of side *a* is 6 cm, side *b* is 8 cm, and side *c* is 10 cm.



3. Copy the table below into your notebook.

	Side	Side Length (cm)	Angle Opposite the Side (°)	Area of Square (cm^2)	Right Triangle? (yes/no)
Triangle 1	*a*	6	37		
	b	8			
	c	10			
Triangle 2	*a*	5			
	b	7			
	c	10			
Triangle 3	*a*	5		25	
	b			144	
	c			169	

4. Measure the angle opposite each side of Triangle 1 with a protractor.



5. In your table, record the angle measures to the nearest degree.

6. Complete the rest of the table for Triangle 1.

7. Repeat the above steps for Triangles 2 and 3 in the table.

Reflect on Your Findings

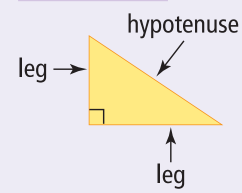
8. a) Which triangles are right triangles? How do you know?
   b) For each right triangle, write an addition statement showing the relationship between the areas of the three squares.
   c) For each right triangle, describe in words the relationship between the side lengths of the triangle.

Literacy Link

Right Triangle

A *right triangle* has a right angle (90°). The right angle may be marked with a small square.

The two shorter sides that form the right angle are called the legs. The longest side is called the **hypotenuse**.



hypotenuse

- the longest side of a right triangle
- the side opposite the right angle

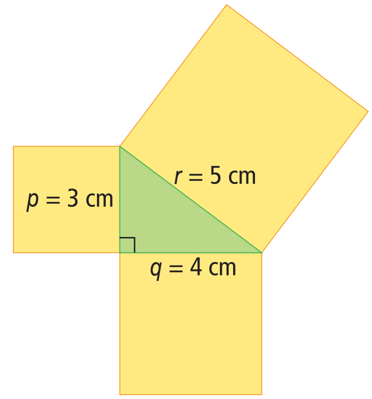
Example 1: Describe Relationships in Right Triangles

a) What is the area of each square?

b) Which side is the hypotenuse of the triangle?

c) Write an addition statement showing the relationship between the areas of the three squares.

d) Describe, using words and symbols, the relationship between the side lengths of the triangle.



Solution

a) $p = 3$ cm, $A = 3^2$, $A = 9$. The area is 9 cm^2.

$q = 4$ cm, $A = 4^2$, $A = 16$. The area is 16 cm^2.

$r = 5$ cm, $A = 5^2$, $A = 25$. The area is 25 cm^2.

This relationship is called the **Pythagorean relationship**.

b) Side r is the hypotenuse.

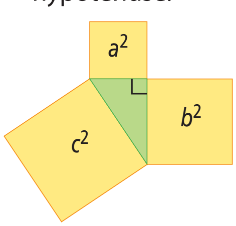
c) $9 + 16 = 25$

d) The sum of the areas of the squares attached to legs p and q equals the area of the square attached to hypotenuse r.

For a right triangle with legs p and q and hypotenuse r, $p^2 + q^2 = r^2$.

Pythagorean relationship

- the relationship between the lengths of the sides of a right triangle
- The sum of the areas of the squares attached to the legs of a right triangle equals the area of the square attached to the hypotenuse.



$a^2 + b^2 = c^2$

Show You Know

The sides of a right triangle are 9 cm, 12 cm, and 15 cm.

a) Sketch a picture of the triangle. Draw a square on each side of the triangle.

b) What is the area of each square?

c) Write an addition statement using the areas of the three squares.

Example 2: Identify a Right Triangle

A triangle has side lengths of 5 cm, 7 cm, and 9 cm.

a) What are the areas of the three squares that can be drawn on the sides of the triangle?

b) Is the triangle a right triangle? Explain your answer.

WWW Web Link

To learn more about the Pythagorean relationship, go to www.mathlinks8.ca and follow the links.

Literacy Link

The symbol ≠ means "is not equal to."

Solution

a) $5 \times 5 = 25$
The area is 25 cm^2.

$7 \times 7 = 49$
The area is 49 cm^2.

$9 \times 9 = 81$
The area is 81 cm^2.

b) Calculate the sum of the areas of the two smaller squares.
$25 + 49 = 74$
The sum of the areas is 74 cm^2. The sum does not equal the area of the large square. 74 $cm^2 \neq$ 81 cm^2
The triangle is not a right triangle.

Show You Know

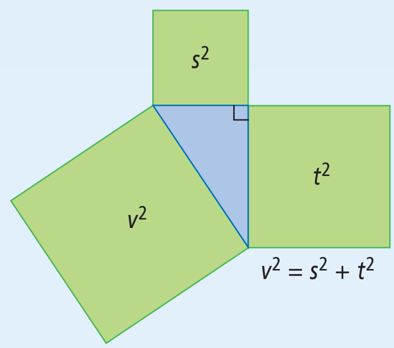
A triangle has side lengths of 12 cm, 16 cm, and 20 cm.

a) What are the areas of the three squares that can be drawn on the sides of the triangle?

b) Is the triangle a right triangle? Explain.

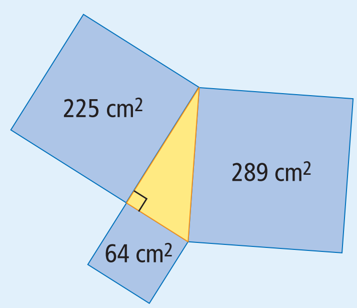
Key Ideas

- In a right triangle, the sum of the areas of the squares attached to the legs equals the area of the square attached to the hypotenuse.
- The Pythagorean relationship states that in a right triangle with sides s, t, and v, where side v is the hypotenuse, $v^2 = s^2 + t^2$.



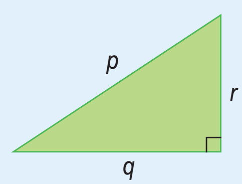
Communicate the Ideas

1. Describe, using words and symbols, the relationship among the areas of the three squares shown.



2. A triangle has side lengths of 7 cm, 11 cm, and 15 cm. Explain how you can determine whether or not it is a right triangle.

3. For the triangle shown, Kendra wrote the Pythagorean relationship as $r^2 = p^2 + q^2$. Is she correct? Explain.

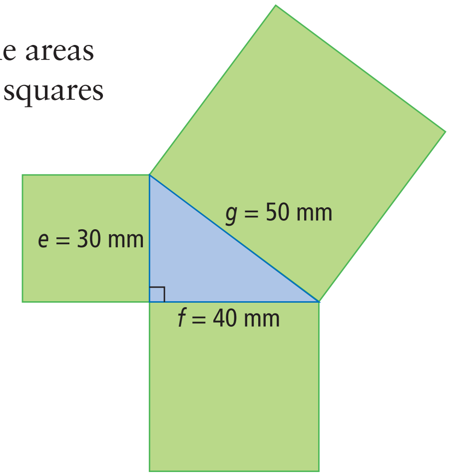


Check Your Understanding

Practise

For help with #4 to #7, refer to Example 1 on page 90.

4. What are the areas of the three squares shown?



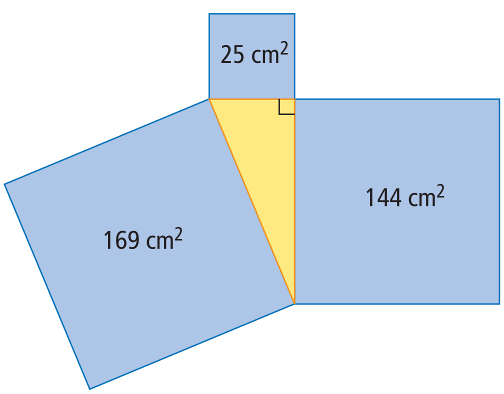
5. A right triangle has side lengths of 40 mm, 75 mm, and 85 mm.

a) Sketch the triangle. Draw a square on each side of the triangle.

b) What are the areas of the three squares?

c) Write an addition statement with the areas of the three squares.

6. a) Write an addition statement using the areas of these three squares.



b) What is the side length of each square?

c) Describe, using words and symbols, the relationship between the side lengths of each square.

7. The sides of a right triangle measure 9 cm, 12 cm, and 15 cm.

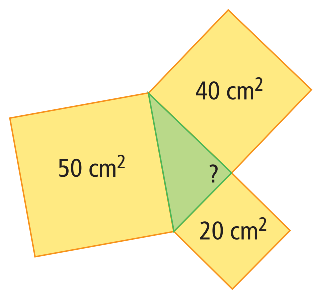
a) What is the area of each square attached to the three sides of the right triangle?

b) Write an addition statement showing the relationship between the areas of the three squares.

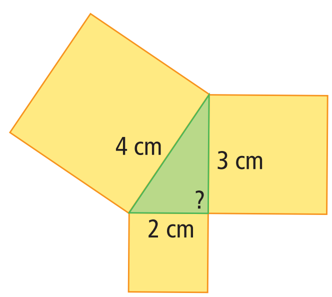
c) Describe, using words and symbols, the relationship between the side lengths of each square.

For help with #8 to #11, refer to Example 2 on pages 90–91.

8. Is the triangle shown a right triangle? Explain your reasoning.



9. a) Calculate the areas of the three squares.



b) Is this triangle a right triangle? Explain.

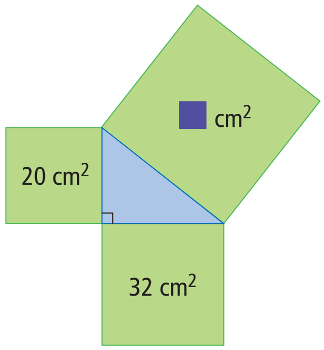
10. A triangle has side lengths of 120 mm, 160 mm, and 200 mm. Is the triangle a right triangle? Explain your reasoning.

11. The side lengths of a triangle are 5 cm, 6 cm, and 8 cm. Determine whether the triangle is a right triangle. Explain.

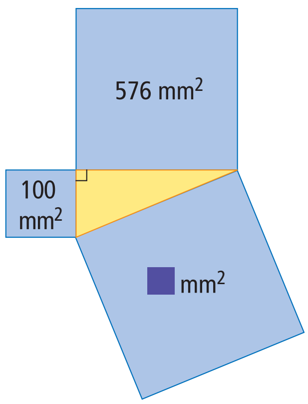
Apply

12. Use the Pythagorean relationship to find the unknown area of each square.

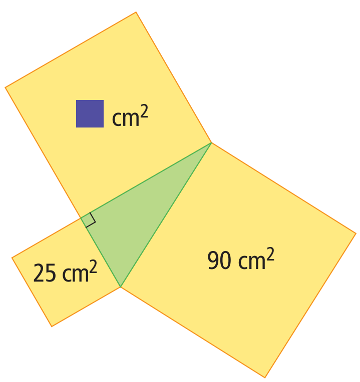
a)



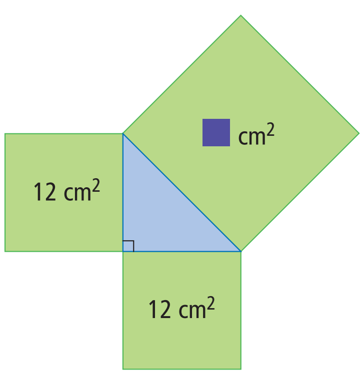
b)



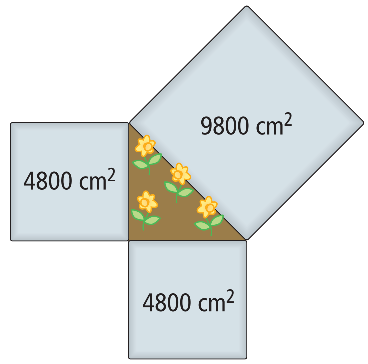
c)



d)



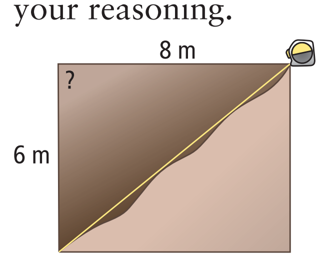
13. A small triangular flower bed has a square stepping stone at each of its sides. Is the flower bed in the shape of a right triangle? Explain your reasoning.



14. Show whether each triangle in the table is a right triangle.

Triangle	Side Lengths (cm)
A	9, 12, 15
B	7, 8, 11
C	7, 24, 25
D	16, 30, 34
E	10, 11, 14

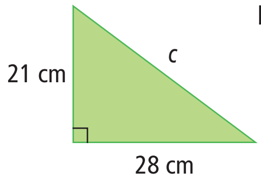
15. Construction workers have begun to dig a hole for a swimming pool. They want to check that the angle they have dug is 90°. They measure the diagonal as shown to be 9.5 m. Is the angle 90°? Explain your reasoning.



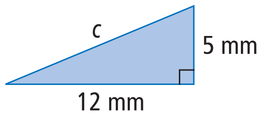
16. Baldeep is building a wooden box for storing coloured pencils. The box will have rectangular sides that are 12 cm wide and 20 cm long. Show how Baldeep can be sure the sides are rectangular, without using a protractor.

17. What is the area of the square that can be drawn on side *c* of each triangle?

a)

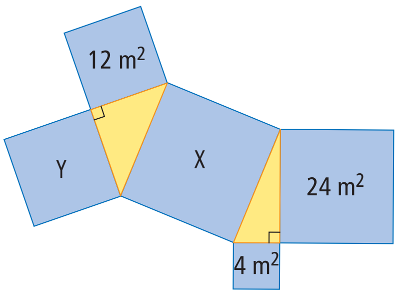


b)



Extend

18. The diagram is made of two right triangles and five squares.

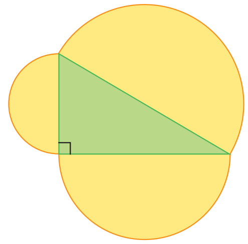


a) What is the area of square X?

b) What is the area of square Y?

19. A right triangle has a square attached to each side. Two of the squares have areas of 10 cm^2 and 15 cm^2. What are possible areas for the third square? Draw a sketch for each solution.

20. A right triangle has sides of 3 cm, 4 cm, and 5 cm. Attached to each side is a semi-circle instead of a square. Describe the relationship between the areas of the semi-circles.



Literacy Link

area of a circle $= \pi \times r^2$

21. An example of a Pythagorean triple is 3, 4, 5.

a) Multiply each number by 2. Show whether the resulting three numbers form a Pythagorean triple.

b) Multiply each number in the triple 3, 4, 5 by a natural number other than 2. Show whether the results form a Pythagorean triple.

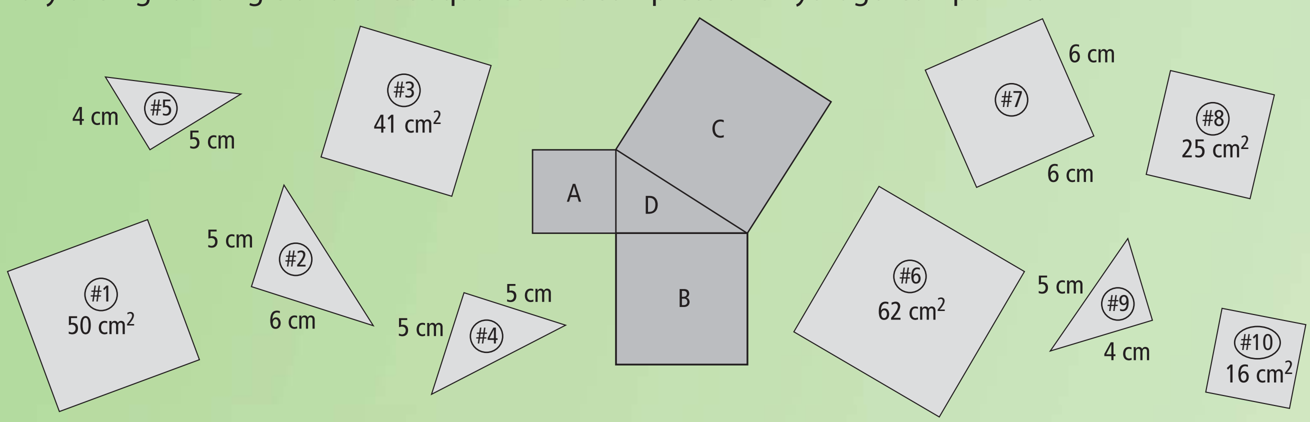
c) Is there any natural number that does not make a Pythagorean triple when 3, 4, 5 are multiplied by it? Explain.

Did You Know?

A Pythagorean triple consists of three whole numbers that form the sides of a right triangle. For example, 3, 4, 5 make a Pythagorean triple because $3^2 + 4^2 = 5^2$.

MATH LINK

Identify the right triangle and three squares that complete this Pythagorean puzzle.



3.3

Estimating Square Roots

Focus on...

After this lesson, you will be able to...

- estimate the square root of a number that is not a perfect square
- identify a number with a square root that is between two given numbers

The picture shows three tatami mats that are used in judo. Can you think of a way to estimate the side length of the middle mat?

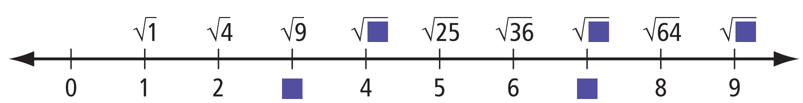
Explore the Math

How do you estimate a square root?

1. What is a reasonable estimate for the area of the middle mat in the picture?

2. What are the side lengths of the smallest and largest mats? Explain how you calculated these dimensions.

3. The number line below shows square roots of perfect squares. Copy the number line into your notebook. Complete the boxes.



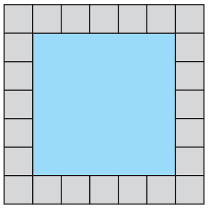
4. Use the number line to estimate the side length for the middle mat. Give your answer to one decimal place.

Reflect on Your Findings

5. a) Compare your estimate of the side length of the middle mat with a classmate's.
   b) Using a calculator, determine the square root of your estimate in #1. Give your answer to the nearest tenth. Compare this approximation to your estimate for the side length.
   c) Explain how you can use perfect squares to estimate a square root.

Example 1: Estimate the Square Root of a Number

Felicity wants to know if a wading pool will fit in a small space in her yard. She must estimate the side length of the square wading pool, which has an area of 7 m^2.



a) What is a reasonable estimate for the side length of the pool? Use perfect squares to estimate. Give your answer to one decimal place.

b) Use a calculator to approximate the side length of the pool, to the nearest tenth of a metre. Compare your estimate in part a) with the calculator's approximate answer.

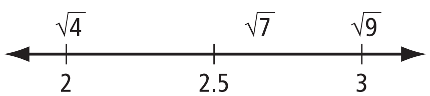
Solution

Strategies
Estimate and Check

a) The side length of the pool is the square root of 7.

The perfect squares on either side of 7 are 4 and 9.

Since 7 is closer to 9, the square root of 7 is closer to the square root of 9.



$\sqrt{9} = 3$
$\sqrt{7}$ will be a bit less than 3.
A reasonable estimate is 2.7 m.

b) Approximate the square root of 7.
C 7 √ 2.645751311
The answer to the nearest tenth of a metre is 2.6 m.
This answer is very close to the estimate of 2.7 m.

This value is an approximation. The decimal portion of the exact answer continues forever. The calculator can display only ten digits.
The square of the approximation shows that it is not an exact answer:
$2.645751311^2 = 6.999999999658218721$
≈ 7

Show You Know

For each of the following, use perfect squares to estimate the square root to one decimal place. Check your answer with a calculator.

a) $\sqrt{18}$ **b)** $\sqrt{23}$ **c)** $\sqrt{35}$

Example 2: Identify a Number With a Square Root Between Two Numbers

a) What is a whole number that has a square root between 6 and 7?

b) How many whole numbers can you find that have a square root between 6 and 7? Show your work.

Solution

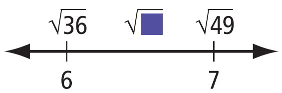
a) Determine the square of 6.

$6^2 = 36$

Determine the square of 7.

$7^2 = 49$

Draw a number line.



Find a value for ■ on the number line.
Choose any whole number between 36 and 49.
One possible whole number is 40.
$\sqrt{40}$ will have a value between 6 and 7.

Check:

C 40 √ 6.32455532

6.32455532 is between 6 and 7.
40 is a possible answer.

b) The possible answers are all of the whole numbers larger than 36 and smaller than 49:

37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48

There are 12 whole numbers that have square roots between 6 and 7.

Strategies

Estimate and Check

Show You Know

a) Identify a whole number with a square root between 8 and 9.

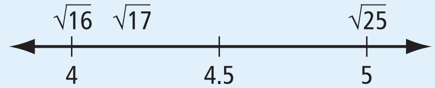
b) How many whole numbers can you find that have a square root between 8 and 9? Show your work.

Key Ideas

- To estimate the square root of a whole number that is not a perfect square,
  - locate the perfect squares on either side of the number
  - calculate the square roots of these two perfect squares
  - estimate based on the position between the two perfect squares

  For example, estimate the square root of 17:

  $\sqrt{17} \approx 4.1$

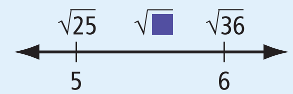


- To identify a whole number that has a square root between two given numbers,
  - determine the perfect squares of the two consecutive whole numbers
  - choose a whole number between the two perfect squares

  For example, identify a whole number that has a square root between 5 and 6:

  $5^2 = 25$ $\qquad$ $6^2 = 36$

  $\sqrt{30}$ will have a value between 5 and 6.



- When using a calculator to find the square root of a natural number that is not a perfect square, the value shown on the calculator is only an approximation.

  C 8 √ 2.828427125

Communicate the Ideas

1. Explain how to estimate $\sqrt{28}$ to one decimal place without using a calculator. Compare your answer with a classmate's.

2. Find a whole number that has a square root between 3 and 4. Explain how you found it.

3. Jason is doing his math homework. He has to find the square root of 10. He presses √ **10** on his calculator and the screen displays 3.16227766. However, when 3.16227766 is multiplied by itself, the answer is not 10. Explain.

Check Your Understanding

Practise

For help with #4 to #5, refer to Example 1 on page 96.

4. Estimate the square root of each number, to one decimal place. Check with a calculator.

a) 72 **b)** 103 **c)** 55

5. Estimate each value, to one decimal place. Check your answer with a calculator.

a) $\sqrt{14}$ **b)** $\sqrt{86}$ **c)** $\sqrt{136}$

For help with #6 to #9, refer to Example 2 on page 97.

6. What is an example of a whole number that has a square root between 9 and 10?

7. Identify a whole number with a square root between 11 and 12.

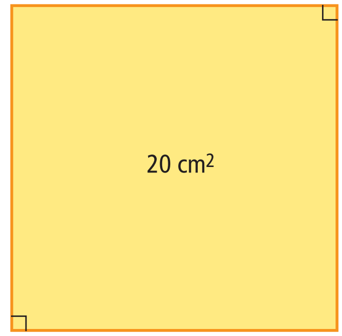
8. Identify all possible whole numbers with a square root larger than 2 and smaller than 3.

9. What are all possible whole numbers that have a square root between 4 and 5?

Apply

10. Kai uses an entire can of paint on a square backdrop for the school play. The label on the can states that one can covers 27 m^2 of wall surface. Estimate the backdrop's side length, to one decimal place.

11. The square has an area of 20 cm^2.



a) Use perfect squares to estimate the side length to one decimal place.

b) Check your answer using a ruler to measure the side of the square. Measure to the nearest tenth of a centimetre.

12. While shopping online, Ji Hun finds a square rug with an area of 11 m^2. He needs to know if it will fit in his 4 m × 5 m bedroom.

a) Estimate the side length of the rug, to one decimal place.

b) Check your estimate with a calculator.

c) Will the rug fit? Explain.

13. Stella is planning an outdoor wedding. She would like a square dance floor with an area of 115 m^2.

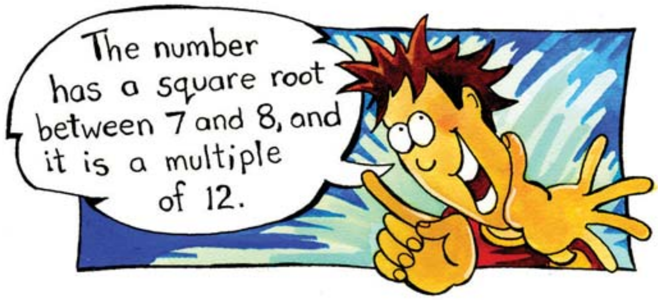
a) Determine the side length of the dance floor, to the nearest tenth of a metre.

b) Stella finds out that the dance floor will be made up of floorboards that each measure 1 m^2. What are the two side lengths the dance floor can have that are closest to what she wants?

c) What are the two square areas for the dance floor that Stella can choose from?

d) Which area will Stella choose? Explain.

14. Alex is thinking of a number.



a) What number could he be thinking of?

b) Is there more than one answer? Explain.

15. Order the following numbers from least to greatest: 7, $\sqrt{46}$, 5.8, $\sqrt{27}$, 6.3.

16. A fitness centre will install a square hot tub in a 6 m × 6 m room. They want the tub to fill no more than 75% of the room's area.

a) What is the maximum area of the hot tub?

b) What dimensions, to a tenth of a metre, will the fitness centre order from the manufacturer? Explain.

17. Carmel wants to mount an 18 cm × 18 cm square picture on a square board that is four times the area of the picture.

a) What is the area of the picture?

b) What is the area of the board?

c) What are the dimensions of the board?

Extend

18. **a)** Evaluate $\sqrt{9}$.

b) Estimate the square root of your answer in part a), to one decimal place.

c) Use a calculator to check your estimate. Express your answer to the nearest hundredth.

d) How close is your estimate in part b) to your calculation in part c)?

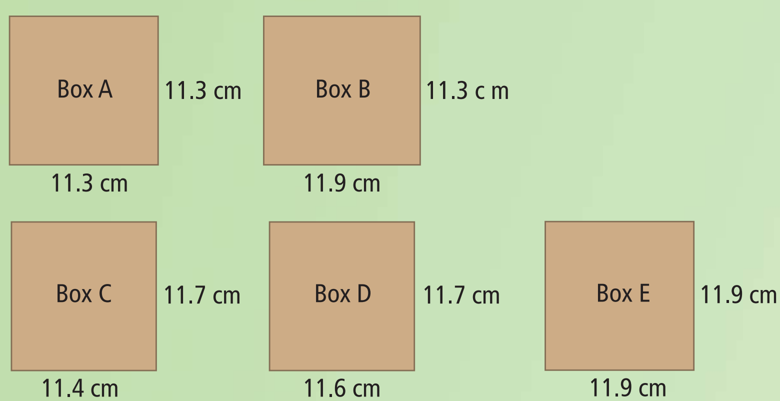
19. Estimate $\sqrt{160\ 100}$. Explain how you determined your estimate.

20. What is the smallest natural number value for n if the solution for $\sqrt{56n}$ is also a natural number?

21. Determine two numbers that have a square root between 326 and 327, are divisible by 100, and are a multiple of 6.

MATH LINK

You have created a mini peg board game called Mind Buster. The square game board has a base area of 134 cm^2. You go to the store to get a box for storing the game. You find five boxes with the base dimensions shown.



a) Identify which boxes can store the game board. Explain.

b) Which box would you choose? Why?

3.4 Using the Pythagorean Relationship

Focus on...

After this lesson, you will be able to...

- ❑ use the Pythagorean relationship to determine the missing side length of a right triangle

A baseball diamond is a square. How could you determine the distance from second base to home plate? How many different strategies can you develop?

Explore the Math

Materials
- centimetre grid paper
- ruler

How do you determine the missing side length of a right triangle?

1. On centimetre grid paper, draw a right triangle.

2. Describe two methods for finding the length of the hypotenuse of a right triangle.

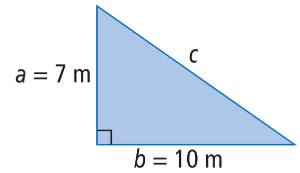
Reflect on Your Findings

3. a) Describe a situation in which one method would be better to use than another.

b) Work with a partner to determine the distance from second base to home plate on a baseball diamond. Share your solution with another pair of classmates.

Example 1: Determine the Length of the Hypotenuse of a Right Triangle

Determine the length of hypotenuse c. Express your answer to the nearest tenth of a metre.



Solution

Strategies
Solve an Equation

Strategies
What other method(s) could you use to solve this problem?

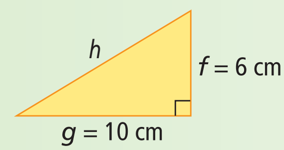
Use the Pythagorean relationship, $c^2 = a^2 + b^2$, where the length of the hypotenuse is c, and the lengths of the legs are a and b.

$$c^2 = 7^2 + 10^2$$
$$c^2 = 49 + 100$$
$$c^2 = 149$$
$$c = \sqrt{149}$$
$$c \approx 12.2$$

The length of the hypotenuse is approximately 12.2 m.

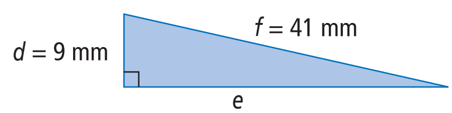
Show You Know

Determine the length of the hypotenuse for the right triangle, to the nearest centimetre.



Example 2: Determine the Length of a Leg of a Right Triangle

What is the length of leg e of the right triangle?



Solution

Strategies
Solve an Equation

Use the Pythagorean relationship, $d^2 + e^2 = f^2$, where the length of the hypotenuse is f, and the lengths of the legs are d and e.

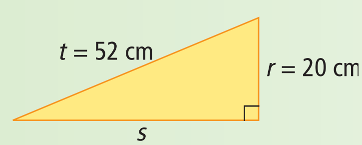
$$9^2 + e^2 = 41^2$$
$$81 + e^2 = 1681$$
$$81 + e^2 - 81 = 1681 - 81$$
$$e^2 = 1600$$
$$e = \sqrt{1600}$$
$$e = 40$$

Why do you subtract 81?

The length of the leg is 40 mm.

Show You Know

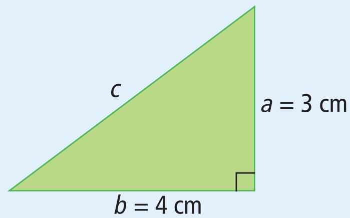
Determine the length of leg s of the right triangle.



Key Ideas

- The Pythagorean relationship can be used to determine the length of the hypotenuse of a right triangle when the lengths of the two legs are known.

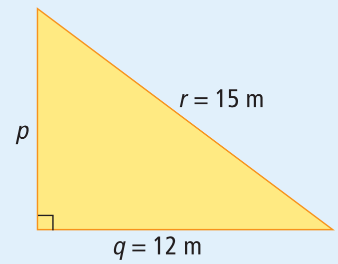
$c^2 = a^2 + b^2$
$c^2 = 3^2 + 4^2$
$c^2 = 9 + 16$
$c^2 = 25$
$c = \sqrt{25}$
$c = 5$



The length of hypotenuse c is 5 cm.

- The Pythagorean relationship can be used to determine the leg length of a right triangle when the lengths of the hypotenuse and the other leg are known.

$p^2 + q^2 = r^2$
$p^2 + 12^2 = 15^2$
$p^2 + 144 = 225$
$p^2 + 144 - 144 = 225 - 144$
$p^2 = 81$
$p = \sqrt{81}$
$p = 9$



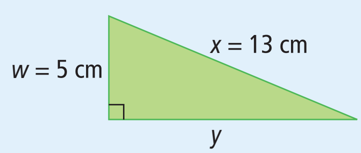
The length of leg p is 9 m.

Communicate the Ideas

1. Jack must determine the missing side length of a triangle. He decides to draw it and then measure it, as shown. Do you agree with the method that Jack is using? Explain.

2. Kira calculated the missing side length of the right triangle.

$y^2 = 5^2 + 13^2$
$y^2 = 25 + 169$
$y^2 = 194$
$y \approx 13.9$
The length of side y is approximately 13.9 cm.



Is Kira correct? If she is correct, explain how you know. If she is incorrect, explain the correct method.

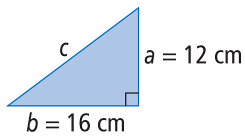
Check Your Understanding

Practise

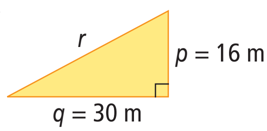
For help with #3 and #4, refer to Example 1 on page 102.

3. Determine the length of each hypotenuse.

a)

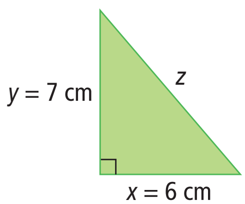


b)



4. What is the length of each hypotenuse? Give your answer to the nearest tenth of a centimetre.

a)



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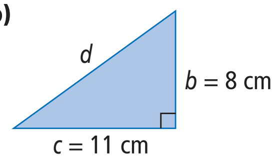
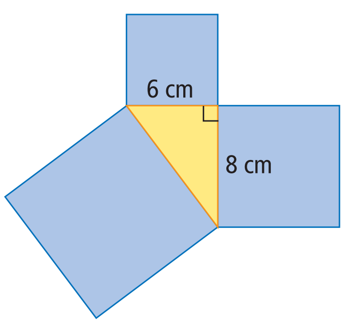


5. **a)** What is the area of each square attached to the legs of the right triangle?



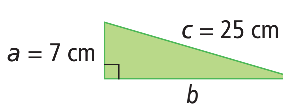
b) What is the area of the square attached to the hypotenuse?

c) What is the length of the hypotenuse?

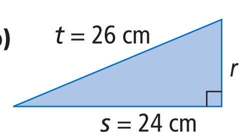
For help with #6 and #7, refer to Example 2 on page 102.

6. Determine the length of the leg for each right triangle.

a)

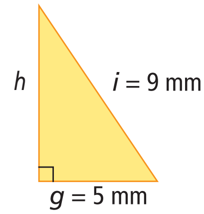


b)

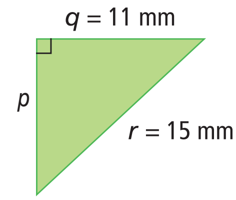


7. What is the missing length of the leg for each triangle? Give your answer to the nearest tenth of a millimetre.

a)

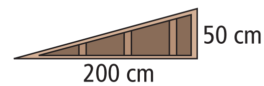


b)

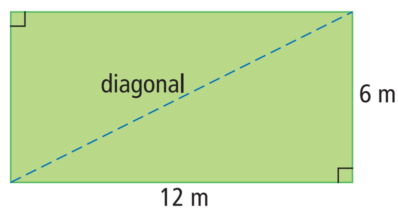


Apply

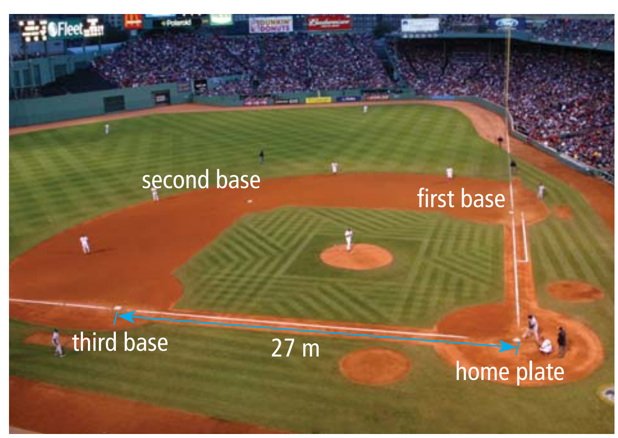
8. The side view of a ramp at a grocery store is in the shape of a right triangle. Determine the length of the ramp, to the nearest centimetre.



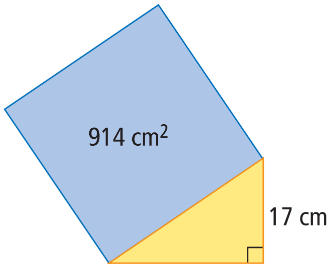
9. Tina wants to construct a path along the diagonal of her yard. What length will the path be? Express your answer to the nearest tenth of a metre.



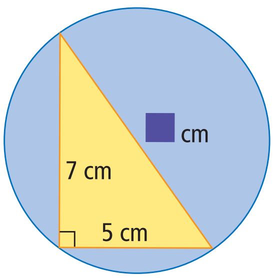
10. What is the minimum distance the player at third base has to throw the ball to get the runner out at first base? Express your answer to the nearest tenth of a metre.



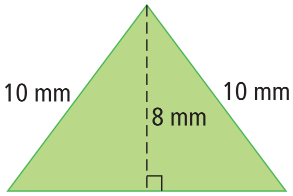
11. The right triangle below has a square attached to its hypotenuse. What is the perimeter of the triangle? Give your answer to the nearest tenth of a centimetre.



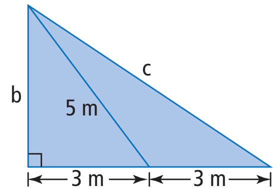
12. The hypotenuse of the triangle cuts the circle in half. What is the diameter of the circle? Express your answer to the nearest tenth of a centimetre.



13. Determine the length of the base of the large triangle. Express your answer to the nearest tenth of a millimetre.

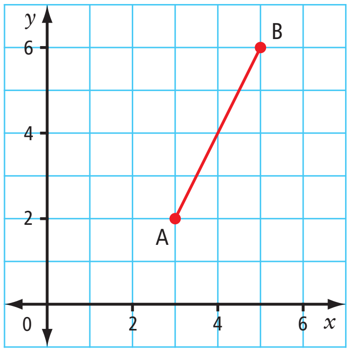


14. What are the lengths of b and c? Write your answer to the nearest tenth of a metre where appropriate.

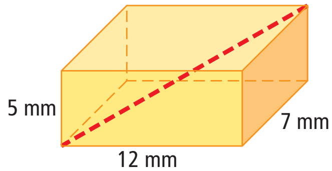


Extend

15. The coordinate grid shown was drawn on centimetre grid paper. What is the length of line segment AB? Express your answer to the nearest tenth of a centimetre.



16. What is the length of the red diagonal in the box? Express your answer to the nearest tenth of a millimetre.



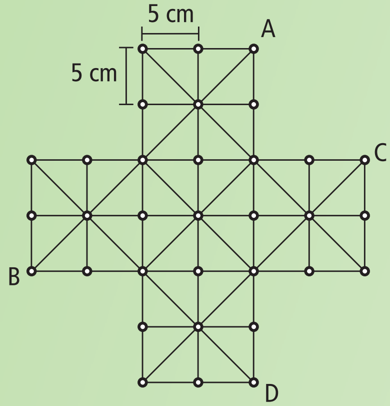
MATH LINK

For each of the following questions, express your answer to the nearest tenth of a centimetre.

a) What is the distance between A and B? Explain.

b) If you have to follow the lines on the game board, what is the shortest distance between C and D?

c) If you do not have to follow the lines on the game board, what is the shortest distance between C and D? Justify your answer.



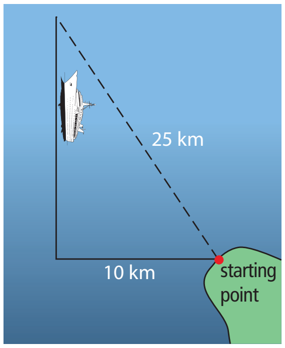
3.5 Applying the Pythagorean Relationship

Focus on...

After this lesson, you will be able to...

- ❑ apply the Pythagorean relationship to solve problems
- ❑ determine distances between objects

A ship leaves the Pacific coast of British Columbia and travels west for 10 km. Then, it turns and travels north. When the ship is 25 km from its starting point, how could you use the Pythagorean relationship to determine the distance the ship travelled north?



Geography Link

North, south, east, and west are directions. On a compass, they are called the cardinal points.

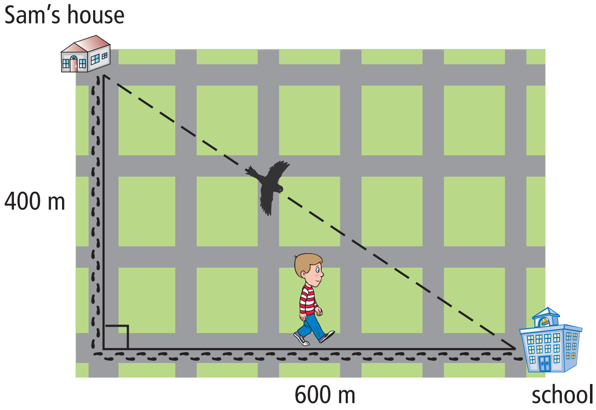
Explore the Math

How can you determine a distance using the Pythagorean relationship?

The diagram shows Sam's trip to school.

1. a) Work with a partner to determine how far his house is from the school.

b) Share your answer with your classmates. Is there more than one possible answer? Explain.



2. a) What do you think the expression "as the crow flies" means?

b) How much farther does Sam travel than the crow? Show your method.

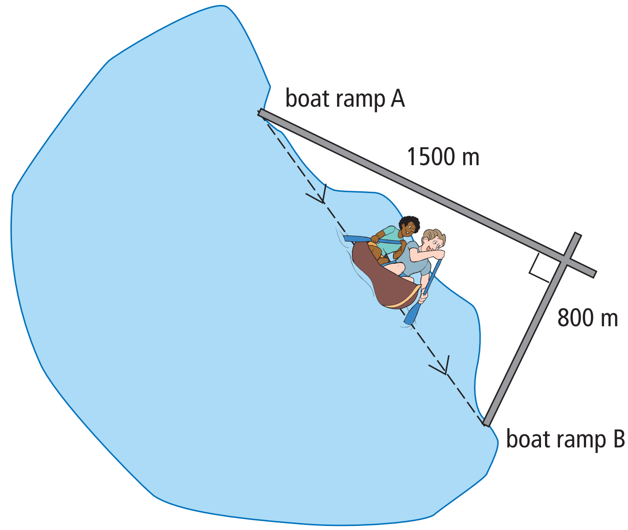
Reflect on Your Findings

3. Why is the path that the crow takes from Sam's house to the school difficult to measure directly?

Example 1: Determine Distances With Right Triangles

a) Anthony and Shalima are canoeing on a lake in Saskatchewan. There are two boat ramps on the lake. How far is it by canoe between the boat ramps?

b) How much farther is it for someone to travel by road from ramp A to ramp B than to canoe between the two ramps?



Solution

a) The two roads leading from the boat ramps make the legs of a right triangle. The distance by canoe is the hypotenuse.

Let d represent the distance by canoe.

Use the Pythagorean relationship.

$d^2 = 1500^2 + 800^2$

$d^2 = 2\,250\,000 + 640\,000$

$d^2 = 2\,890\,000$

$d = \sqrt{2\,890\,000}$

$d = 1700$

The distance by canoe is 1700 m.

Strategies
Solve an Equation

Strategies
What other method could you use to solve this problem?

b) Determine the total distance by road between the boat ramps.

$1500 + 800 = 2300$

The total distance by road is 2300 m.

Determine the difference between the two distances.

$2300 - 1700 = 600$

It is 600 m farther to travel by road than by canoe between the boat ramps.

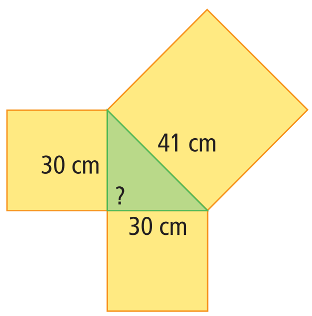
Show You Know

Refer to the opening paragraph and picture on page 106. A ship leaves the Pacific coast of British Columbia and travels west for 10 km. Then, it turns and travels north. If the boat is 25 km from its starting point, what distance did it travel north? Give your answer to the nearest tenth of a kilometre.

Example 2: Verify a Right Angle Triangle

Danelle is trying to install a corner shelf in her bedroom. Since the shelf does not fit properly, she thinks the two walls in her bedroom do not meet at a right angle. She measures a length of 30 cm along the base of each wall away from the corner. Then, she measures the hypotenuse to be 41 cm. Do the walls meet at a right angle? Explain.

Solution



Strategies

Draw a Diagram

Strategies

What other method could you use to solve this problem?

Use the Pythagorean relationship to determine whether the triangle is a right triangle.

Determine whether the sum of the areas of the two smaller squares equals the area of the large square.

Left Side:

$30^2 + 30^2 = 900 + 900$

$= 1800$

The sum of the areas of the two smaller squares is 1800 cm^2.

Right Side:

$41^2 = 1681$

The area of the large square is 1681 cm^2.

$$1800 \text{ cm}^2 \neq 1681 \text{ cm}^2$$

The triangle is not a right triangle. The walls do not meet at a right angle.

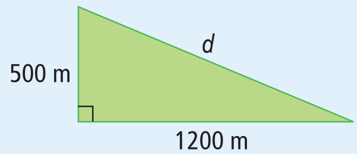
Show You Know

A construction company is digging a rectangular foundation with a width of 17 m and a length of 20 m. To check that a corner is a right angle, a worker measures the diagonal length, which is 26.25 m. Is the corner a right angle? Explain.

Key Ideas

- The Pythagorean relationship can be used to determine distances that might be difficult or impossible to measure.

$d^2 = 500^2 + 1200^2$
$d^2 = 250\,000 + 1\,440\,000$
$d^2 = 1\,690\,000$
$d = \sqrt{1\,690\,000}$
$d = 1300$

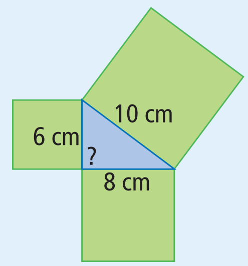


The hypotenuse is 1300 m.

- The Pythagorean relationship can be used to show if a triangle is a right triangle.

Left Side:
$6^2 + 8^2 = 36 + 64$
$= 100$
The sum of the areas of the two smaller squares is 100 cm^2.

Right Side:
$10^2 = 100$
The area of the large square is 100 cm^2.



$100 \text{ cm}^2 = 100 \text{ cm}^2$

The triangle is a right triangle.

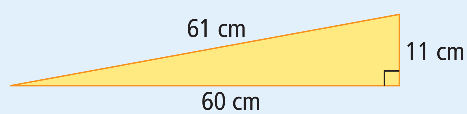
Communicate the Ideas

1. Use an example from real life to explain how you can apply the Pythagorean relationship to calculate distance.

2. Ilana used the following method to determine whether the diagram shows a right triangle.

Left Side:
The large square is 61 cm.

Right Side:
$11 + 60 = 71$
The two smaller squares are 71 cm.



61 cm ≠ 71 cm
The triangle is not a right triangle.

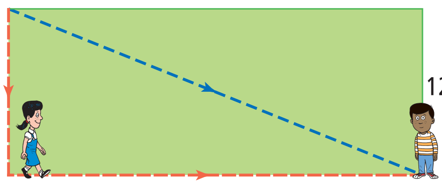
Is Ilana's method correct? If it is correct, explain how you know. If it is incorrect, explain the method llana should use.

Check Your Understanding

Practise

For help with #3 and #4, refer to Example 1 on page 107.

3. Walter walks across a rectangular field in a diagonal line. Maria walks around two sides of the field. They meet at the opposite corner.

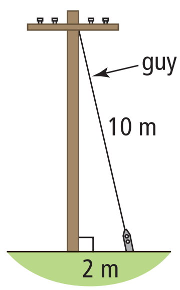


a) How far did Maria walk?

b) How far did Walter walk? Express your answer to the nearest metre.

c) Who walked farther? By how much?

4. Find the height of the pole where the guy wire is attached, to the nearest tenth of a metre.



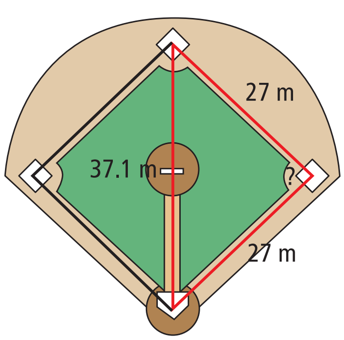
For help with #5 and #6, refer to Example 2 on page 108

5. Martin measured a rectangle and wrote:

Width: 9 cm Length: 22 cm Diagonal: 23.8 cm

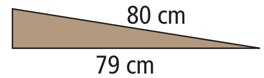
Could these measurements form a rectangle? Justify your answer.

6. You are asked to check the design plans for a baseball diamond. Is the triangle a right triangle? Explain.

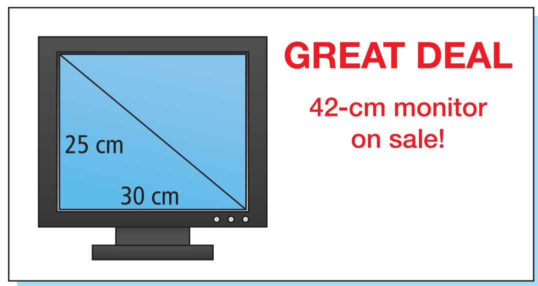


Apply

7. What is the height of the wheelchair ramp? Give your answer to the nearest tenth of a centimetre.



8. Shahriar knows that the size of a computer monitor is based on the length of the diagonal of the screen. He thinks that the diagonal is not as large as the ad says. Is he correct? Explain.

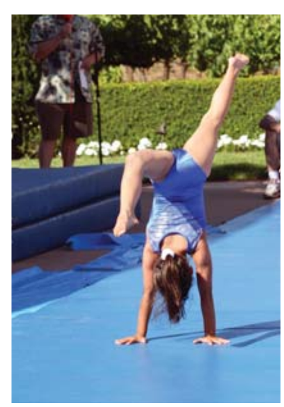


9. A checkerboard is made of 64 small squares that each have a dimension of 3 cm × 3 cm. The 64 small squares are arranged in eight rows of eight.

a) What is the length of the diagonal of a small square? Give your answer to the nearest tenth of a centimetre.

b) What is the total length of the diagonal of the board? Give your answer to the nearest centimetre.

10. A gymnast requires a distance of 16 m for her tumbling routine. If the gymnast is competing on a 12 m × 12 m square mat, does she have enough room to do her routine safely? Explain your answer.



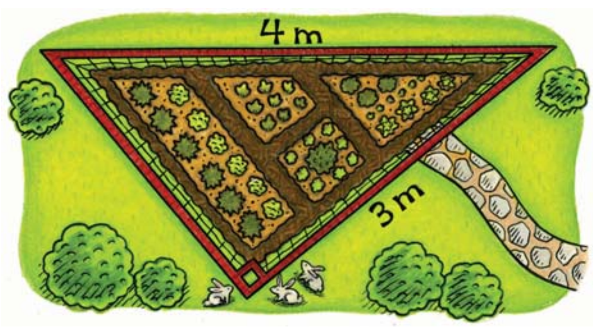
11. Johan has a 300-cm ladder that he leans up against a wall. The safety sticker on the side of the ladder shows that the bottom must be placed between 70 cm and 110 cm away from the wall. What are the minimum distance and maximum distance up the wall that the ladder can reach? Give your answers to the nearest tenth of a centimetre.

Extend

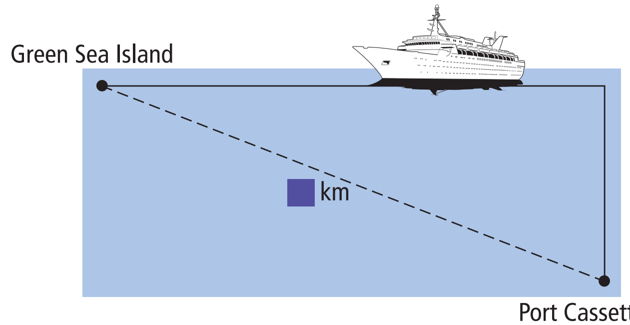
12. Sarah has a vegetable garden in the shape of a right triangle. She wants to put fencing all around it to keep the rabbits away.

a) What total length of fencing does she need? Give your answer to the nearest hundredth of a metre.

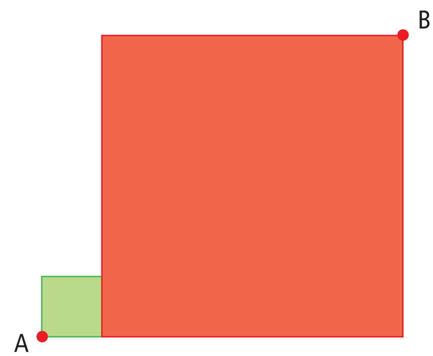
b) If fencing costs \$2/m, what will be the total cost of the fencing?



13. A cruise ship travels from Port Cassett north at a speed of 34 km/h for 2.5 h. Then it turns 90° and travels west at 30 km/h for 7.3 h. When it reaches Green Sea Island, how far is the ship from Port Cassett? Express your answer to the nearest kilometre.



14. The red square has a perimeter of 40 mm and the green square has an area of 4 mm^2. What is the shortest distance between A and B? Give your answer to the nearest tenth of a millimetre.



MATH LINK

The diagram shows the rough plans for a board game designed for a toy manufacturer. The board is composed of a square and four identical right triangles. Complete the plans by answering the following questions. Give your answers to the nearest tenth of a centimetre where appropriate.

a) If the central square has an area of 225 cm^2, what is the perimeter of the game board? Show how you know.

b) The game will be packaged in a box with a square base. Determine the minimum diagonal length of the base of the box.