9.2a Quadratic Inequalities in One Variable

There are two methods to solve quadratic inequalities in one variable.

1. By graphing
2. By finding the roots and using test points

Method 1: Solve by graphing

Ex.1 Solve $x^2 - 10x + 16 \leq 0$

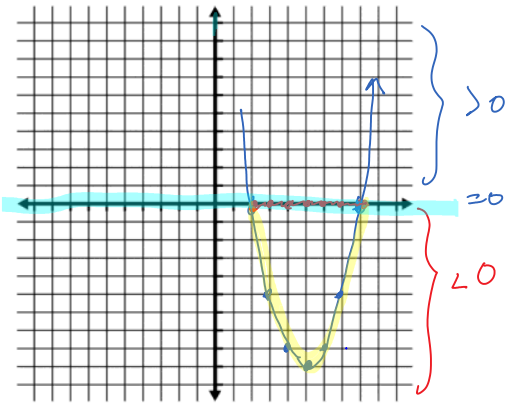
$(x-2)(x-8) \leq 0$

$x = 2, 8$

Graph $y = x^2 - 10x + 16$

$y = (x^2 - 10x + 25) + 16 - 25$

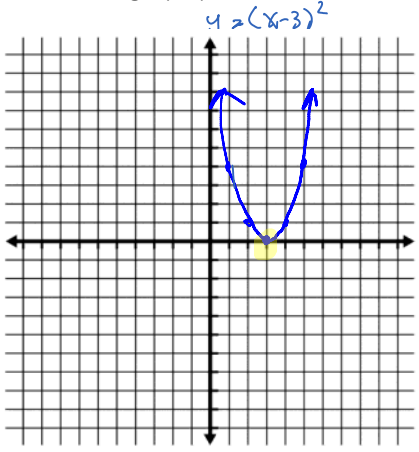
$y = (x-5)^2 - 9$



Which part is below the x-axis?
[y-values are negative]

$2 \leq x \leq 8$

Ex. 2 Given the graph $y = x^2 - 6x + 9$, what is the solution to:



a) $x^2 - 6x + 9 \geq 0$
$x \in \mathbb{R}$

b) $x^2 - 6x + 9 > 0$
$x \in \mathbb{R}, x \neq 3$

c) $x^2 - 6x + 9 \leq 0$
$x = 3$

d) $x^2 - 6x + 9 < 0$
no solution

Method 2: Solve by finding roots and using test points

Ex. 3 Solve $2x^2 + x \leq 6$

$2x^2 + x - 6 \leq 0$

-12 } $4, -3$
1

$2x^2 + 4x - 3x - 6 \leq 0$

$2x(x+2) - 3(x+2) \leq 0$

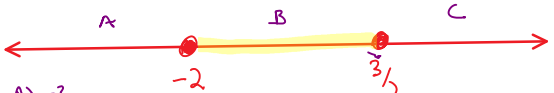
$(2x-3)(x+2) \leq 0$

$x = 3/2 \quad x = -2$

① move all terms to one side

② find zeros by factoring or using quad formula

③ Place roots on # line + test regions. (sketch or parabola)



A) -3
$2(-3)^2 + (-3) - 6 \leq 0$
$18 - 3 - 6 \leq 0$
$9 \leq 0$

B) 0
$-6 \leq 0$ ✓

c) 2
$2(2)^2 + 2 - 6 \leq 0$
$8 + 2 - 6 \leq 0$

$2(-3)^2+(-3)-6 \le 0$
$18-3-6 \le 0$
$9 \le 0$ x

-1
$-6 \le 0$ ✓

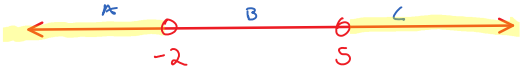
-2
$2(2)^2+2-6 \le 0$
$8+2-6 \le 0$
$4 \le 0$ x

$-2 \le x \le 3/2$

Ex. 4 Solve $-x^2 + 3x + 10 < 0$

$x^2-3x-10 > 0$

$-(x^2-3x-10) < 0$
$-(x-5)(x+2) < 0$
$x = 5, -2$



A) -3
$-(-3)^2+3(-3)+10 < 0$
$-9-9+10 < 0$
$-8 < 0$
True

B) 0
$10 < 0$
x

C) 6
$-(6)^2+3(6)+10 < 0$
$-36+18+10 < 0$
True

$x < -2$ or $x > 5$

Ex. 5 Solve $x^2 - 4x > 10$

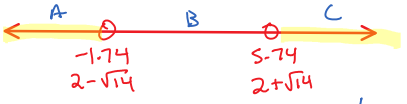
$x^2-4x-10 > 0$

$x = \frac{4 \pm \sqrt{16-4(1)(-10)}}{2}$

$x = \frac{4 \pm \sqrt{56}}{2}$

$x = \frac{4 \pm 2\sqrt{14}}{2}$

$x = 2 \pm \sqrt{14}$ → 5.74, −1.74



A) -2
$(-2)^2-4(-2)-10 > 0$
$4+8-10 > 0$
✓

B) 0
$-10 > 0$

C) 6
$6^2-4(6)-10 > 0$
$36-24-10 > 0$
$2 > 0$
✓

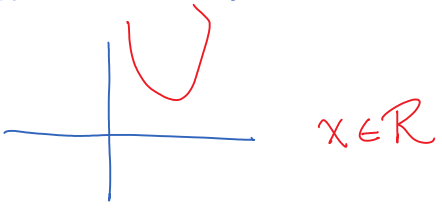
$x < 2-\sqrt{14}$ or $x > 2+\sqrt{14}$

Ex. 6 Solve $x^2 - 4x > -10$

$x^2-4x+10 > 0$

$x = \frac{4 \pm \sqrt{16-4(1)(10)}}{2} = \frac{4 \pm \sqrt{-24}}{2}$

$b^2-4ac < 0$ so no real roots!
$x \neq 0$



$x \in R$

note... 1) if $b^2-4ac = 0$, then one root on y-axis

2) if $b^2-4ac < 0$, then no roots... careful, you will need to sketch to determine if $x \in R$ or there is no solution.

Assignment: p484 # 1, 2, 3ac, 4, 7ab, 8ac, 9ac

don't explain

4d) answer $-2-\frac{\sqrt{6}}{2} \leq x \leq -2+\frac{\sqrt{6}}{2}$

text is missing this neg.