There are two methods to solve a system of equations algebraically:
i. Substitution Method
ii. Elimination Method

Remember an (x,y) pair is a single solution to a system.

I. Substitution Method

- Isolate one variable in one of the equations
- Substitute the expression into the other equation and solve for the remaining variable
- Substitute that value into one of the equations to find the value of the other variable.

*the variables should make both equations true

Ex. 1 Solve $3x + y = -9 \rightarrow y = -3x - 9$
$4x^2 - x + y = -9 \rightarrow y = -4x^2 + x - 9$

$-3x - 9 = -4x^2 + x - 9$
$4x^2 - 4x = 0$
$4x(x-1) = 0$
$x = 0 \quad x = 1$

Logic
A = B
A = C
∴ B = C

if $x=0$
$3(0) + y = -9$
$y = -9$
$\boxed{(0,-9)}$

or

if $x=1$
$3(1) + y = -9$
$3 + y = -9$
$y = -12$
$\boxed{(1,-12)}$

two solutions

Ex. 2 Solve $6x^2 - x - y = -1 \rightarrow 6x^2 - x + 1 = y$
$4x^2 - 4x - y = -6$

replace y in other eq'n

$4x^2 - 4x - (6x^2 - x + 1) = -6$
$4x^2 - 4x - 6x^2 + x - 1 = -6$
$-2x^2 - 3x - 1 = -6$
$0 = 2x^2 + 3x - 5$
$0 = (x-1)(2x+5)$
$x = 1$ or $x = -5/2$ or -2.5

Factor:
$-10, 3$; $+5, -2$
$2x^2 + 5x - 2x - 5$
$x(2x+5) - 1(2x+5)$

if $x = 1$
$6(1)^2 - (1) - y = -1$
$6 - 1 - y = -1$
$5 - y = -1$
$6 = y$
$\boxed{(1,6)}$

or

if $x = -2.5$
$6(-2.5)^2 - (-2.5) - y = -1$
$37.5 + 2.5 - y = -1$
$40 - y = -1$
$41 = y$
$\boxed{(-2.5, 41)}$

II. Elimination Method

- Rearrange terms so that like terms line up
- Create opposite coefficients for the variable that occurs only once by multiplying one or both equations by a constant
- Add the equations together to eliminate one variable and solve for the remaining variable
- Substitute that value into one of the equations to find the remaining unknown variable

*the variables should make both equations true when substituted in.

Ex. Solve $5x^2 + 3y = -3 - x$ → $[5x^2 + 3y + x = -3]\,2$

$2x^2 - x = -4 - 2y$ → $[2x^2 + 2y - x = -4]\,3$

Need to eliminate, so make 6

$$10x^2 + 6y + 2x = -6$$
$$-\ (6x^2 + 6y - 3x = -12)$$
$$4x^2 + 5x = 6$$
$$4x^2 + 5x - 6 = 0$$
$$(4x-3)(x+2) = 0$$
$$x = 3/4 \text{ or } x = -2$$

if $x = \frac{3}{4}$

$$2(3/4)^2 - (3/4) = -4 - 2y$$
$$2(9/16) - 3/4 = -4 - 2y$$
$$\left(\frac{9}{8} - \frac{3}{4} = -4 - 2y\right)8$$
$$9 - 6 = -32 - 16y$$
$$3 = -32 - 16y$$
$$35 = -16y$$
$$-\frac{35}{16} = y$$
$$\boxed{(3/4, -35/16)}$$

if $x = -2$

$$2(-2)^2 - (-2) = -4 - 2y$$
$$8 + 2 = -4 - 2y$$
$$10 = -4 - 2y$$
$$14 = -2y$$
$$-7 = y$$
$$\boxed{(-2, -7)}$$

Ex. 4 Is (2, -5) a solution to the following system of equations? ← Balance both?

$4x^2 - 7y^2 = -159$

$-3x^2 + 5y^2 = 112$

Balance?

$$4(2)^2 - 7(-5)^2 = -159$$
$$16 - 175 = -159$$
$$-159 = -159$$

yes balanced.

Balance?

$$-3(2)^2 + 5(-5)^2 = 112$$
$$-3(4) + 5(25) = 112$$
$$-12 + 125 = 112$$
$$113 = 112$$

No, not balanced.

∴ Not a Solution of the system

Ex. 5 Use any method to solve: (Explain why you chose to use that method)

*note, have to eliminate "h" to solve

$d^2 - 2d + 3h = 9$

$5d^2 - 10d + h = 0$ → $h = 10d - 5d^2$

Substitution?

$$d^2 - 2d + 3(10d - 5d^2) = 9$$
$$d^2 - 2d + 30d - 15d^2 = 9$$
$$-14d^2 + 28d = 9$$
$$0 = 14d^2 - 28d + 9$$

OR Elimination

$$d^2 - 2d + 3h = 9$$
$$3\times \quad -\ (15d^2 - 30d + 3h = 0)$$
$$-14d^2 + 28d = 9$$
$$0 = 14d^2 - 28d + 9$$

continue

Using Quad. formula...

$$d = \frac{28 \pm \sqrt{(-28)^2 - 4(14)(9)}}{2(14)} = \frac{28 \pm \sqrt{784 - 504}}{28}$$

$$= \frac{28 \pm \sqrt{280}}{} \nearrow d = 1.6$$

28 $\to d = 0.4$

if $d = 1.6$

$h = 10(1.6) - 5(1.6)^2$

$h = 16 - 12.8$

$h = 3.2$

$\boxed{(1.6, 3.2)}$

if $d = 0.4$

$h = 10(0.4) - 5(0.4)^2$

$h = 4 - 0.8$

$h = 3.2$

$\boxed{(0.4, 3.2)}$

Assignment: p451 # 1, 2, 3abce, 4abe, 5a, 6, 7