7.4a Linear Reciprocal Functions

$\frac{0}{1}$ $\frac{y}{x}$ $\frac{a}{b} \times \frac{b}{a} = 1$

What is a reciprocal?

A product of a number and its reciprocal always equal to 1.

The reciprocal of n $= \dfrac{1}{n}$, if n $\neq$ 0 because n $\times \dfrac{1}{n} = 1$

1
$\frac{1}{1000}$ $\frac{1}{200}$

Find the reciprocal of the following numbers:

10	5	1	$\frac{1}{2}$	0	$-\frac{3}{4}$	-1	-20	-100
$\frac{1}{10}$	$\frac{1}{5}$	1	2	undefined	$\frac{-4}{3}$	-1	$-\frac{1}{20}$	$-\frac{1}{100}$

- What do you notice when you take the reciprocal of large pos/neg numbers?

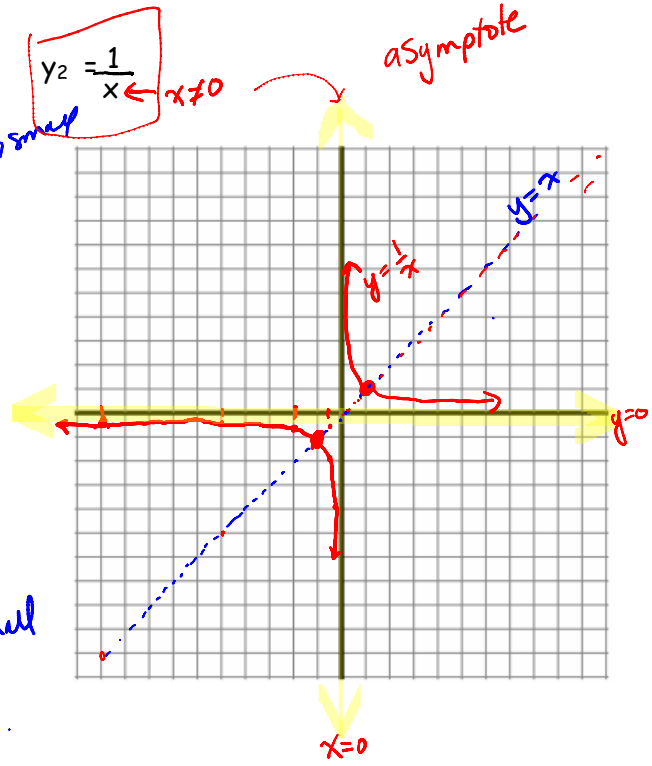- What do you notice when you take the reciprocal of small numbers $[-1 < x < 1]$?

<u>Comparing Graphs of a funtion and its reciprocal</u>

Let's compare the two graphs $y_1 = x$ and $y_2 = \dfrac{1}{x}$ $\leftarrow x \neq 0$

asymptote

x	y_1	y_2
-10	-10	$\frac{1}{-10}$
-5	-5	$\frac{1}{-5}$
-2	-2	$-\frac{1}{2}$
-1	-1	-1
-0.5	$-\frac{1}{2}$	-2
-0.25	$-\frac{1}{4}$	-4
-0.01	$-\frac{1}{100}$	-100
0	0	no reciprocal.
0.01	$\frac{1}{100}$	100
0.25	$\frac{1}{4}$	4
0.5	$\frac{1}{2}$	2
1	1	1
2	2	$\frac{1}{2}$
5	5	$\frac{1}{5}$
10	10	$\frac{1}{10}$

Big → small

Big

Big

small

$y = \frac{1}{x}$ $y = x$ $y = 0$

$x = 0$

In general, to graph a reciprocal function $\dfrac{1}{f(x)}$:

1. Graph $y = f(x)$
2. Draw the vertical asymptote(s). ← *The value of x that makes the denominator zero*

   This is the x-intercept because $f(x) = 0$, and reciprocal of zero is undefined.
3. Draw the horizontal asymptote.

   The value y approaches as $|x|$ approaches infinity. $(y=0)$
4. Label the invariant points.

   When $f(x) = 1$ or -1
5. Use asymptotes and invariant points to sketch.

   (Big → Small) When $f(x) > 1$, the reciprocal function approaches the horizontal asymptote

   (Small → Big) When $0 < f(x) < 1$, the reciprocal function approaches the vertical asymptote

---

Example 1:

Given $f(x) = \frac{1}{2}x - 4$

a) Determine $\dfrac{1}{f(x)}$

$$y_2 = \frac{1}{\frac{1}{2}x - 4} \quad \swarrow \quad x \neq 8$$

$$\frac{1}{2}x - 4 = 0$$
$$\frac{1}{2}x = 4$$
$$x = 8$$

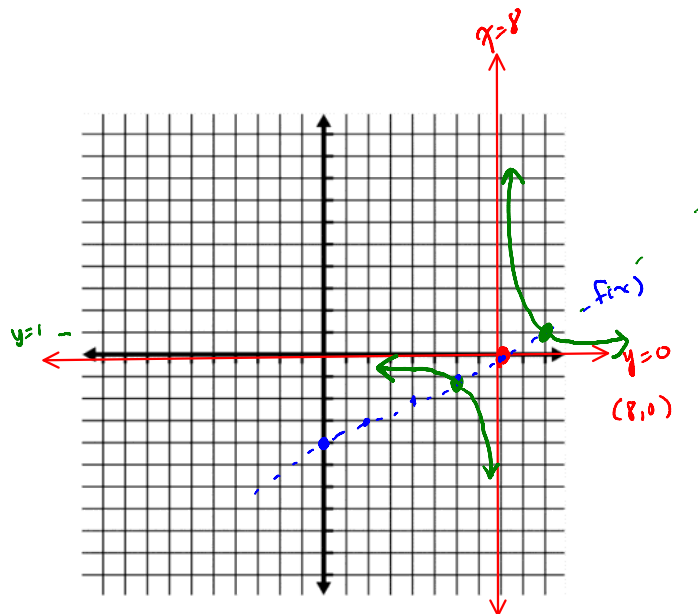b) Determine the horizontal and vertical asymptotes.

c) Graph

1. Graph $f(x)$

2. Asymptotes
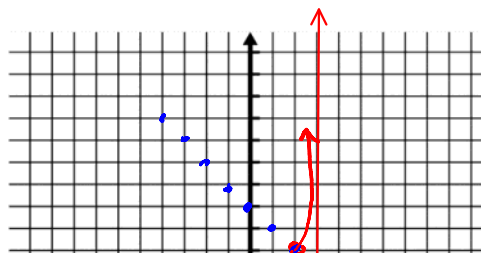
   * the x-int of $f(x)$

3. Invariant points

$x = 8$

$y = 1$

$y = 0$

$(8, 0)$

$f(x)$

4. Sketch $\dfrac{1}{f(x)}$ going through invariant points and approaching asymptotes

---

Example 2:

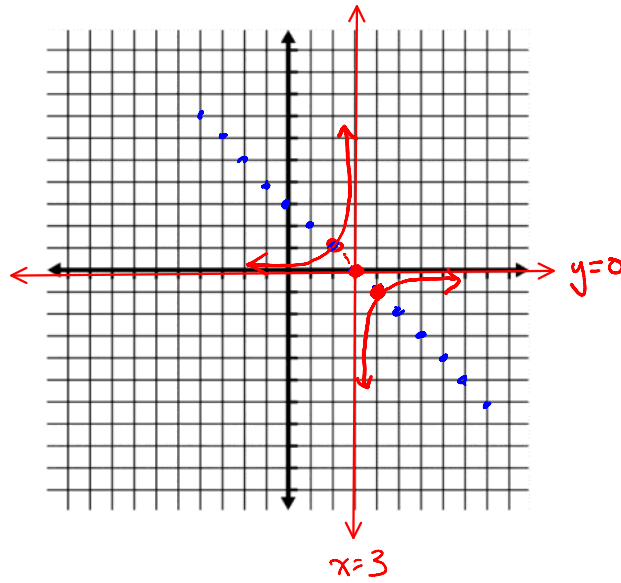Graph $y = \dfrac{1}{3-x}$

1) Graph $y = 3 - x$

or

Graph $y = \dfrac{1}{3-x}$

1) Graph $y = 3-x$
   or
   $y = -x+3$

2) Asymptotes?
   $y=0$ (horizontal)

   $x=3$ (vertical)
   $\uparrow$
   x-int.

3) Invariant pts
   where $y = \pm 1$

$y=0$

$x=3$

Assignment: p403 #1ab, 2ab, 3ab, 4, 5ab, 6a, 7, 9ad