7.3 Absolute Value Equations

To solve absolute value equations:

1. Isolate the absolute value expression
2. Create two equations.
   i) The expression inside the absolute value function is positive.
   ii) The expression inside the absolute value function is negative.
3. Solve each equation and check for extraneous roots.

$$|x| = 5$$

$x = 5$ or $x = -5$

$x^2 = 9$
$x = \pm 9$
$(x+2)^2 = \pm 3$
$x+2 = \pm 3$

$2|x| - 3 = 7$
$2|x| = 10$
$|x| = 5$

Ex. 1 Solve and show how each solution can be found graphically.

$y_1 = y_2$ The dist. between a # and 5 is 9.

a) $|x - 5| = 9$

$y = |x-5|$
$y = 9$

$x - 5 = 9$ $x - 5 = -9$
$x = 14$ $x = -4$
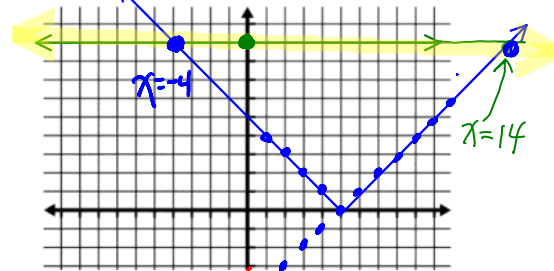
check: $|14-5| = 9$
$|9| = 9$ ✓

check: $|-4-5| = 9$
$|-9| = 9$ ✓

$x = -4$
$x = 14$

b) $|x + 5| = 4x - 1$

$x + 5 = 4x - 1$ $x + 5 = -(4x-1)$
$6 = 3x$ $x + 5 = -4x + 1$
$2 = x$ $5x = -4$
                        $x = -0.8$

$y = |x+5|$ $y = 4x - 1$
$y = x+5$
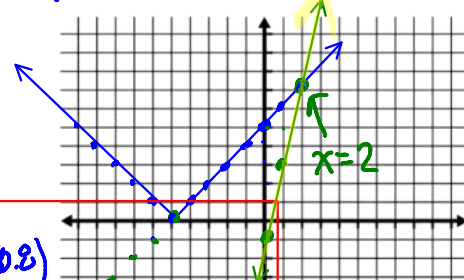
check: $|2+5| = 4(2) - 1$
$|7| = 7$ ✓

check: $|-0.8+5| = 4(-0.8)$
$|+4.2| = -4.2$
NO, can't be neg

$x = 2$

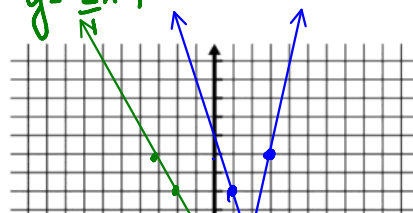c) $|4x - 7| = -2x - 1$

$4x - 7 = -2x - 1$ or $4x - 7 = -(-2x-1)$
$6x = 6$ $4x - 7 = 2x + 1$
$x = 1$ $2x = +8$
                        $x = +4$

$y = |4x-7|$
$y = -2x - 1$

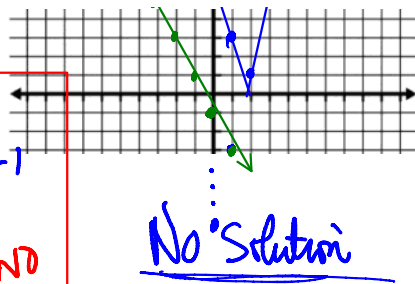check:

Ch7 Absolute Value Page 1

check: 1

$$|4(1)-7| = -2(1)-1$$

$$|-3| = -3 \quad \text{NO}$$

Ex. 2 Solve

check +4

$$|4(4)-7| = -2(4)-1$$

$$|9| = -9 \quad \text{NO}$$

No Solution

a) $\dfrac{5|2x+3|}{5} = \dfrac{30}{5}$

$$|2x+3| = 6$$

$$2x+3 = 6 \qquad\qquad 2x+3 = -6$$
$$2x = 3 \qquad\qquad\quad 2x = -9$$
$$x = 1.5 \qquad\qquad\quad x = -4.5$$
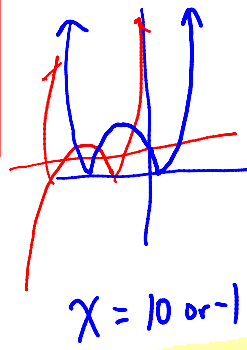
$$\boxed{x = 1.5 \text{ or } -4.5}$$

Check:
$$5|2(1.5)+3| = 30$$
$$5(6) = 30 \checkmark$$

Check:
$$5|2(-4.5)+3| = 30$$
$$5(6) = 30 \checkmark$$

b) $|x - 10| = x^2 - 10x$

$$x - 10 = x^2 - 10x \qquad\qquad x - 10 = -(x^2 - 10x)$$
$$0 = x^2 - 11x + 10 \qquad\qquad x - 10 = -x^2 + 10x$$
$$0 = (x-10)(x-1) \qquad\qquad x^2 - 9x - 10 = 0$$
$$x = 10 \text{ or } \cancel{1} \qquad\qquad (x-10)(x+1) = 0$$
$$\qquad\qquad\qquad\qquad x = 10 \text{ or } -1$$

$$x = 10 \text{ or } -1$$

Check 10:
$$|10-10| = 100-100$$
$$\checkmark$$

Check 1:
$$|1-10| = 1 - 10$$
$$|-9| = -9 \quad \cancel{}$$

Check -1:
$$|-1-10| = 1+10$$
$$|-11| = 11 \quad \checkmark$$

Ex. 3 A program controls the amount of chips packaged into a bag. If the ideal mass is 143 grams with an error of 4 grams. Between what masses can the bag of chips be? Determine the absolute value equation.

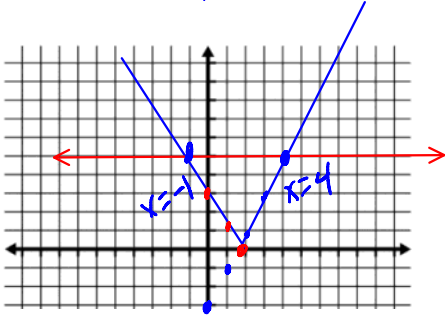$$143 + 4 = \boxed{147g}$$
$$143 - 4 = \boxed{139g}$$

$$|M - 143| = 4$$

↑ dist from the mean

<u>Reminder:</u>

When asked to **solve by graphing**.....

1 Graph the left side $y_1 = |2x - 3|$

$$\left| 2x - 3 \right| = 5$$

2. Graph the right side $y_2 = 5$
3. Find the points of intersection.
   The values of x that balance the equation.
   (where $y_1 = y_2$)

$$y_1 = \left| 2x - 3 \right|$$

$$y_2 = 5$$

don't need to graph.

Assignment: p389 #2ac,4,5ace, 6abe, 7,9,10,12,22,17*

Quiz next class!