

6.4b Rational Equations

Solving Problems using Rational Equations

Reminders:

Two consecutive numbers: $x, x + 1$

Two consecutive even/odd numbers: $x, x + 2$

Two numbers that have a sum of 15: $x, 15 - x$

Two numbers that have a difference of 8: $x, x + 8$ or $x, x - 8$

Example 1

The sum of the reciprocals of two consecutive odd integers is $\frac{8}{15}$.

What are the numbers?

1st #: x

2nd #: $x + 2$

$$\left[\frac{1}{x} + \frac{1}{x+2} = \frac{8}{15} \right] (x)(x+2)(15)$$

$$(1)(x+2)(15) + (1)(x)(15) = (8)(x)(x+2)$$

$$15x + 30 + 15x = 8x^2 + 16x$$

$$8x^2 - 14x - 30 = 0$$

$$4x^2 - 7x - 15 = 0$$

$$(4x+5)(x-3) = 0$$

$$x = -\frac{5}{4} \text{ (neglect negative value) or } x = 3$$

\therefore The numbers are 3 and 5

Example 2

If Carl can paint a room in four hours and Jane can paint the same room in three hours, how long will it take if they paint the room together?

Let x = the time they will take when they paint the room together

	time	per hour	per " x " hours
Carl	4	$\frac{1}{4}$	$\frac{x}{4}$
Jane	3	$\frac{1}{3}$	$\frac{x}{3}$
Together	x	$\frac{1}{x}$	1

$$\frac{x}{4} + \frac{x}{3} = 1$$

$$3x + 4x = 12$$

$$7x = 12$$

$$x = \frac{12}{7} \text{ hr}$$

$$x = 1 \text{ hour } \frac{5}{7} \times 60 \text{ minutes}$$

$$x = \mathbf{1 \text{ hour } 43 \text{ min}}$$

\therefore It will take 1 hour 43 minutes if they paint the room together

Example 3

A speedboat can travel 108 km downstream in the same time it can travel 78 km upstream. If the current in the river is 10km/h, what is the speed of the boat in still water?

Let x = the speed of the boat in water

	d	s	t
Downstream (faster)	108	$x + 10$	$\frac{108}{x + 10}$
Upstream (slower)	78	$x - 10$	$\frac{78}{x - 10}$

$$\frac{108}{x + 10} = \frac{78}{x - 10}$$

$$108(x - 10) = 78(x + 10)$$

$$108x - 1080 = 78x + 780$$

$$30x = 1860$$

$$x = \mathbf{62 \text{ km/h}}$$

\therefore The speed of the boat in water is 62km/h.

Example 4

Anna and Blair have two different delivery routes. Anna's is 80km and Blair's route is 100km. Blair travels 10km/h faster than Anna, and finishes 10 minutes earlier. What are the speeds of each driver?

Let $x = \text{Anna's speed}$

	d	s	t
Anna	80	x	$\frac{80}{x}$
Blair	100	$x + 10$	$\frac{100}{x + 10}$

$$t_A - t_b = 10 \text{min}$$

$$\frac{80}{x} - \frac{100}{x + 10} = 10 \times \frac{1}{60}$$

$$\left[\frac{80}{x} - \frac{100}{x + 10} = \frac{1}{6} \right] (x)(x + 10)(6)$$

$$(80)(x + 10)(6) - (100)(x)(6) = (1)(x)(x + 10)$$

$$480x + 4800 - 600x = x^2 + 10x$$

$$x^2 + 130x - 4800 = 0$$

$$(x + 160)(x - 30) = 0$$

$$x = -160(\text{neglect the negative}) \text{ or } x = 30$$

\therefore The Anna's speed is 30km/hr and the Blair's speed is 40km/hr.

Assignment: p.348 #3bc,8,9,11,12,14,15,17,18,13*