6.4a Rational Equations
Solving Rational Equations

To solve rational equations:

1. Factor each denominator
2. Multiply both sides of the equation by the denominator's LCM.
3. Solve the linear or quadratic equation
4. Identify NPVs from the original equation, and check your answer.

* Cross-Multiply when each side contains one fraction!

Examples:

1. $\frac{5}{x+2} = \frac{4}{x-1}$ (cross-multiply)

$$5(x-1) = 4(x+2)$$
$$5x - 5 = 4x + 8$$
$$x = 13$$

NPVs?
$x + 2 \neq 0$
$x \neq -2$
$x - 1 \neq 0$
$x \neq 1$

2. $\frac{4x-3}{2x+1} = \frac{2x+1}{x-4}$ (cross-multiply)

$$(4x-3)(x-4) = (2x+1)(2x+1)$$
$$4x^2 - 19x + 12 = 4x^2 + 4x + 1$$
$$-23x = -11$$
$$x = \frac{11}{23}$$

NPVs?
$2x + 1 \neq 0$
$x \neq -\frac{1}{2}$
$x - 4 \neq 0$
$x \neq 4$

3. $\frac{5}{n-3} = \frac{2}{3-n}$ (cross-multiply)

$$5(3-n) = 2(n-3)$$
$$15 - 5n = 2n - 6$$
$$-7n = -21$$
$$n = 3$$
No Solution

NPVs?
$n - 3 \neq 0$
$x \neq 3$
$3 - n \neq 0$
$x \neq 3$

4. $\frac{4}{a} = \frac{1}{2} + \frac{3}{5a}$

$$\left[\frac{4}{a} = \frac{1}{2} + \frac{3}{5a}\right](a)(2)(5)$$
$$(4)(2)(5) = (1)(a)(5) + (3)(2)$$
$$40 = 5a + 6$$
$$5a = 34$$
$$a = \frac{34}{5}$$

NPVs?
$a \neq 0$
$2 \neq 0$
No restriction
$5a \neq 0$
$a \neq 0$

5. $\frac{2m+3}{m+3} = \frac{1}{2} + \frac{m+1}{m-1}$

$$\left[\frac{2m+3}{m+3} = \frac{1}{2} + \frac{m+1}{m-1}\right](m+3)(2)(m-1)$$
$$(2m+3)(2)(m-1) = (1)(m+3)(m-1) + (m+1)(m+3)(2)$$
$$(4m+6)(m-1) = (m^2 - m + 3m - 3) + (m+1)(2m+6)$$
$$4m^2 - 4m + 6m - 6 = m^2 - m + 3m - 3 + 2m^2 + 6m + 2m + 6$$
$$4m^2 + 2m - 6 = 3m^2 + 10m + 3$$
$$m^2 - 8m - 9 = 0$$
$$(m-9)(m+1) = 0$$
$$m = 9 \text{ or } m = -1$$

NPVs?
$m + 3 \neq 0$
$m \neq -3$
$m - 1 \neq 0$
$m \neq 1$