Chapter 6(I) Rational Expressions
6.3b Adding & Subtracting Rational Expressions

Completely factor each denominator before determining the LCM.

Example

a) $\frac{2}{x^2+4x+3} - \frac{3x}{x^2-1} = \frac{2}{(x+1)(x+3)} \times \frac{(x-1)}{(x-1)} - \frac{3x}{(x+1)(x-1)} \times \frac{(x+3)}{(x+3)} = \frac{(2x-2)-(3x^2+9x)}{(x+1)(x+3)(x-1)}$

$= \frac{-3x^2-7x-2}{(x+1)(x+3)(x-1)} = \boxed{\frac{-(3x+1)(x+2)}{(x+1)(x+3)(x-1)}}$

b) $\frac{5x}{x^2+5x} - \frac{4}{25-x^2} = \frac{5x}{x(x+5)} - \frac{4}{(5-x)(5+x)} = \frac{5x}{x(x+5)} \times \frac{(x-5)}{(x-5)} + \frac{4}{(x-5)(x+5)} \times \frac{x}{x}$

$= \frac{5x^2-25x+4x}{x(x+5)(x-5)} = \frac{5x^2-21x}{x(x+5)(x-5)} = \frac{x(5x-21)}{x(x+5)(x-5)} = \boxed{\frac{5x-21}{(x+5)(x-5)}}$

c) $\frac{x^2-9}{x^2-x-12} - \frac{x^2-5x-14}{x^2-4x-21} = \frac{(x+3)(x-3)}{(x-4)(x+3)} \times \frac{(x+3)}{(x+3)} - \frac{(x-7)(x+2)}{(x-7)(x+3)} \times \frac{(x-4)}{(x-4)}$

$= \frac{(x^2-9)-(x^2-4x+2x-8)}{(x-4)(x+3)} = \boxed{\frac{2x-1}{(x-4)(x+3)}}$

d) $\frac{\frac{3}{2x}+2}{\frac{4}{3x}-1} = \frac{\frac{3}{2x}+\frac{2}{1}\times\frac{2x}{2x}}{\frac{4}{3x}-\frac{1}{1}\times\frac{3x}{3x}} = \frac{\frac{3+4x}{2x}}{\frac{4-3x}{3x}} = \frac{3x(3+4x)}{2x(4-3x)} = \boxed{\frac{3(4x+3)}{2(4-3x)}}$

e) $\frac{2-\frac{4}{y}}{y-\frac{4}{y}} = \frac{\frac{2}{1}\times\frac{y}{y}-\frac{4}{y}}{\frac{y}{1}\times\frac{y}{y}-\frac{4}{y}} = \frac{\frac{2y-4}{y}}{\frac{y^2-4}{y}} = \frac{\frac{2(y-2)}{y}}{\frac{(y+2)(y-2)}{y}} = \frac{2y(y-2)}{y(y+2)(y-2)} = \boxed{\frac{2}{y+2}}$

One person runs a 1000m race at a speed of n m/sec. Another person runs the same at n + 5 m/sec. How much longer does it take for the first person to run the race?

$\frac{1000}{n}\times\frac{(n+5)}{(n+5)} - \frac{1000}{n+5}\times\frac{(n)}{(n)} = \frac{1000n+5000-1000n}{n(n+5)} = \boxed{\frac{5000}{n(n+5)} \text{ sec}}$

Assignment: p337 #6-9, 10ab, 13, 15ad