

Chapter 6(I) Rational Expressions
6.1a Simplifying Rational Expressions

Rational Expression

A rational expression is the ratio of two polynomials.

Ex. $\frac{x^2+5}{x+2}$

It is undefined for any value of the variable that makes the denominator equal to 0.

Ex. Determine the restriction on each rational expression.

a) $\frac{4}{3x}$
 $3x \neq 0$
 $x \neq 0$

b) $\frac{4x}{3x+1}$
 $3x+1 \neq 0$
 $3x \neq -1$
 $x \neq -\frac{1}{3}$

c) $\frac{7}{x(5x-3)}$
 $x(5x-3) \neq 0$
 $x \neq 0$ $5x-3 \neq 0$
 $x \neq \frac{3}{5}$

d) $\frac{4}{x^3-9x}$
 $x^3-9x \neq 0$
 $x(x^2-9) \neq 0$
 $x(x+3)(x-3) \neq 0$
 $x \neq 0, -3, 3$

e) $\frac{3x^2+2x+1}{x^2-5x+6}$
 $x^2-5x+6 \neq 0$
 $(x-2)(x-3) \neq 0$
 $x \neq 2, 3$

f) $\frac{3x}{8}$
 $8 \neq 0$ True
 \therefore No restriction

g) $\frac{1}{x^2+3}$
 $x^2+3 \neq 0$
 $\sqrt{x^2} \neq \sqrt{-3}$
 No solution
 \therefore No restriction

h) $\frac{1}{x^2-25}$
 $x^2-25 \neq 0$
 $(x+5)(x-5) \neq 0$
 $x \neq -5, 5$

Simplifying Rational Expressions

a) $\frac{6x^4y^3}{-18x^5y^2} = -\frac{1}{3}xy$

b) $\frac{-45a^3b^4c^2}{90a^2bc^6} = -\frac{1}{2} \frac{abc^2}{c^4}$

$$c) \frac{3x^2 - 6x}{3x} = \frac{\cancel{3x}(x-2)}{\cancel{3x}} = \boxed{x-2}$$

$$d) \frac{-4x^5 - 12x^3}{-4x^2} = \frac{\cancel{-4x^3}(x^2+3)}{\cancel{-4x^2}} = \boxed{x(x^2+3)}$$

$$e) \frac{16-4a}{32-8a} = \frac{\cancel{4}(4-a)}{\cancel{8}(4-a)} = \boxed{\frac{1}{2}}$$

$$f) \frac{3x-3y}{9xy-9x^2} = \frac{\cancel{3}(x-y)}{\cancel{9x}(y-x)} = \boxed{\frac{-1}{3x}}$$

$$g) \frac{2-x}{x^2-x-2} = \frac{\cancel{2-x}}{\cancel{(x-2)}(x+1)} = \boxed{\frac{-1}{x+1}}$$

$$h) \frac{x^2-6x-7}{x^2-4x-5} = \frac{(x-7)\cancel{(x+1)}}{\cancel{(x-5)}(x+1)} = \boxed{\frac{x-7}{x-5}}$$

$$i) \frac{\cancel{(x-9)}\cancel{(x+1)}}{\cancel{(x+1)}\cancel{(9-x)}} = \boxed{-1}$$

$$j) \frac{\cancel{(x-4)}\cancel{(7-x)}(3+x)}{(x+5)\cancel{(4-x)}\cancel{(x-7)}} = \boxed{\frac{-(3+x)}{x+5}}$$