Restrictions ?

$$\frac{\sqrt{x}}{\sqrt{x}-1}$$

$x \geq 0$

$\rightarrow \sqrt{x}-1 \neq 0$

$\sqrt{x} \neq 1$

$x \neq 1$

$x \geq 0, \ x \neq 1$

A **radical equation** is an equation with radicals that have variables in the radicands.

$$\begin{bmatrix} \text{A radical expression does not have an} \\ \text{equal sign, so it can only be simplified.} \end{bmatrix}$$

Radical equations:

a) $3-\sqrt{2x-1}=4$ b) $-8+\sqrt{-7x}=3$ c) $x-\sqrt{7-2x}=-7$

State the restrictions on the variable in the above radical equations

$2x-1 \geq 0$
$2x \geq 1$
$x \geq \frac{1}{2}$

$\frac{-7x}{-7} \geq \frac{0}{-7}$
$x \leq 0$

$7-2x \geq 0$
$\frac{-2x \geq -7}{-2} \quad \frac{-7}{-2}$
$x \leq \frac{7}{2}$

Steps to solving radical equations:

1. Identify any restrictions on the variable
2. Solve the equation by isolating the radical, and then variable
*3. Check answers for possible extraneous roots

Extraneous root: a number obtained in solving an eq'n that does not satisfy the initial restrictions on the variable (false root)

Examples.
Solve:

a) $\left(\sqrt{x+1}\right)^2=(2)^2$; $x+1 \geq 0$
$x \geq -1$

$x+1=4$

$x=3$

check: $\sqrt{3+1}=2$ ✓

$\left(\sqrt{a}+\sqrt{b}\right)^2=0$
$\left(\sqrt{a}+\sqrt{b}\right)\left(\sqrt{a}+\sqrt{b}\right)$

b) $\left(2\sqrt{x}\right)^2=\left(4\sqrt{5}\right)^2$; $x \geq 0$

$4x=16(5)$

$4x=80$

$x=20$ $\sqrt{16\cdot5}$ ✓

check: $\sqrt{3+1} = 2$ ✓

$x = 20$

check $\sqrt{480} \overset{\checkmark}{=} 2\sqrt{20} = \overset{\text{4}\sqrt{5}}{\boxed{4\sqrt{5}}} \rightarrow \sqrt{16\cdot5}$ ✓
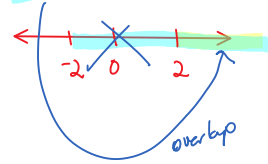
c) $\sqrt{x+5} - 3 = 7$, $x+5 \geq 0$

$\sqrt{x+5} = 7+3$ $\qquad x \geq -5$

$(\sqrt{x+5})^2 = (10)^2$

$x+5 = 100$

check $\qquad x = 95$

$\sqrt{95+5} - 3 = 7$ ✓

d) $\sqrt{2x-4} - \sqrt{x+2} = 0$, $2x-4 \geq 0$ **and** $x+2 \geq 0$

$(\sqrt{2x-4})^2 = (\sqrt{x+2})^2$ $\quad 2x \geq 4$ $\qquad x \geq -2$

$\qquad\qquad\qquad\qquad\quad \boxed{x \geq 2}$

$2x-4 = x+2$

$2x - x = 2+4$

$x = 6$

overlap

check $\sqrt{12-4} - \sqrt{6+2} = 0$ ✓

e) $\sqrt{x-2} - x = -4$, $x-2 \geq 0$

$(\sqrt{x-2})^2 = (x-4)^2$ $\qquad x \geq 2$

$x-2 = x^2 - 8x + 16$

$0 = x^2 - 9x + 18$

$0 = (x-6)(x-3)$

$\boxed{x = 6}, \cancel{3}$

Check 6

$\sqrt{4} - 6 = -4$ ✓

check 3

$\sqrt{1} - 3 = -4$

$1 - 3 = -4$ ✗

$x^2 = 1$

$x = \pm\sqrt{1}$

$x = \pm 1$

f) $\sqrt{3x-7} + \sqrt{2x+5} = 0$

$(\sqrt{3x-7})^2 = (-\sqrt{2x+5})^2$

$3x-7 \geq 0$ **and** $2x+5 \geq 0$

$3x \geq 7$ $\qquad\qquad 2x \geq -5$

$\boxed{x \geq \tfrac{7}{3}}$ $\qquad\qquad x \geq -\tfrac{5}{2}$

$3x-7 = 2x+5$

$3x - 2x = 5+7$

$x = \cancel{12}$

$\sqrt{36-7} + \sqrt{24+5} = 0$

$\underbrace{\phantom{\sqrt{36-7}}}_{\text{pos}} + \underbrace{\phantom{\sqrt{24+5}}}_{\text{pos}} = 0$
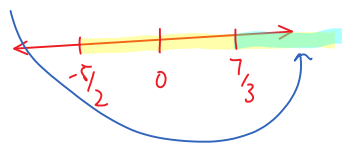
No Solution

Assignment: p300 #3,4,5,6,9abc,10a,13,14,19*